A Rational-Pragmatic Account of Communicative Demonstrations

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Abstract

Theory of mind enables an observer to interpret others' behavior in terms of unobservable beliefs, desires, intentions, feelings, and expectations about the world. This also empowers the person whose behavior is being observed: By intelligently modifying her actions, she can influence the mental representations that an observer ascribes to her, and by extension, what the observer comes to believe about the world. That is, she can engage in intentionally *communicative demonstrations*. Here, we develop a computational account of generating and interpreting communicative demonstrations by explicitly distinguishing between two interacting types of intentions. *Object-directed* intentions aim to control states of the physical environment, whereas *belief-directed* intentions aim to influence an observer's mental representations. This formulation provides a number of theoretical insights. In particular, our framework (1) extends existing formal models of pragmatics and pedagogy to the setting of value-guided decision-making, (2) captures how people modify their intentional behavior to *show* what they know about the reward or causal structure of an environment, and (3) helps explain data on infant and child imitation in terms of differential attribution to adult demonstrators' object-directed and belief-directed intentions. Additionally, our analysis of belief-directed intentionality and mentalizing helps shed light on the socio-cognitive mechanisms that underlie distinctly human forms of communication, culture, and sociality.

Keywords: communication, problem solving, pragmatics, planning, social learning

A Rational-Pragmatic Account of Communicative Demonstrations

Introduction

Demonstration is a powerful force in human social learning: We often acquire new skills by observing others execute them. How does this work? Naively, one might think the best approach is for the demonstrator to faithfully demonstrate the skill just as they would perform it alone, and for the observer to interpret the faithful demonstration as such. It is perhaps surprising, then, that this is far from typical. Rather, demonstrations more often follow the exaggerated behavior of a mime who must communicate what he is doing by actions alone. In fact, the celebrated mime Marcell Marceau was once approached by a trained cellist who scolded Marceau for acting out cello-playing with inaccurate and exaggerated motions. The two of them worked together to create a new act that faithfully reproduced the precise motions of cello playing, but their efforts were wasted: The new routine resembled a person sawing a piece of wood (Ringnalda, 1994). Somehow, Marceau's exaggerations and embellishments communicated the playing of a cello more faithfully than the actual movements of a cellist.

What is the difference between doing and demonstrating—that is, between performing a structured action and conveying the structure to others through performance? Although intertwined, these behaviors are not identical. Our aim is to better characterize their relationship, and thus to explain how people teach with and learn from demonstration.

Teaching and learning by demonstration are routine for humans. *Communicative demonstrations* occur when we coordinate (Clark, 2005), cooperate (Jordan, Hoffman, Bloom, & Rand, 2016), establish the meanings of novel signs (Scott-Phillips, Kirby, & Ritchie, 2009), and control low-level motor behaviors in interactive settings (Wolpert, Doya, & Kawato, 2003). Developmental psychologists, especially, emphasize the importance of communicative demonstrations. These enable infants and children to learn a range of behaviors and representations, including action types, subgoals, tool functions, causal structure, and normative concepts (Brugger, Lariviere, Mumme, & Bushnell, 2007; Southgate, Chevallier, & Csibra, 2009; Király, Csibra, & Gergely, 2013; Hernik & Csibra, 2015; Buchsbaum, Gopnik, Griffiths, & Shafto, 2011; Butler, Schmidt, Bürgel, & Tomasello, 2015; Sage & Baldwin, 2011; Hoehl, Zettersten, Schleihauf, Grätz, & Pauen, 2014).

Many prior treatments of this topic emphasize that theory of mind—the capacity to reason about behavior in terms of mental states (Premack & Woodruff, 1978; Dennett, 1987)—plays a key role in interpreting communicative demonstrations. Imagine watching someone play an actual cello. Based on how they hold the instrument, move their fingers, and produce sounds, you could infer that they have goals like playing a certain melody or beliefs like that their cello is in tune. These types of inferences involve an actor's mental states that are *about the world* and include reasoning about goals (Gergely, Nádasdy, Csibra, & Bíró, 1995), action costs (Jara-Ettinger, Gweon, Tenenbaum, & Schulz, 2015), or false beliefs about the locations of objects (Southgate, Senju, & Csibra, 2007).

Yet, these prior applications of theory of mind often fail to explain what differentiates demonstrating from mere doing. More specifically, this way of using theory of mind restricts itself to asking, "What task is the demonstrator trying to do, and why?". In other words, it exclusively reasons about the *object-directed* intentions of the actor. While there is no question that object-directed mentalizing is a ubiquitous and essential element of observational learning, it alone is explanatorily limited for several reasons.

First, doing and demonstrating can be radically—even unrecognizably—different. This is because demonstrators do not just have object-directed intentions, they also have *belief-directed* intentions that are directed towards the observer's mental states. Object-directed mentalizing treats doing and demonstrating as essentially the same by assuming they result only from object-directed mental states, when in fact, they may not. For instance, when miming the cello, one does not just want to mimic cello-playing motions, but rather wants to express the idea of cello-playing to an audience. Thus, to meaningfully distinguish between the two, an observer needs to also engage in belief-directed mentalizing, asking herself, "What is this demonstrator trying to convey to me? What does she want me to *believe*?"

Second, and relatedly, infants and children learn differently from identical behaviors based solely on whether the demonstrator has previously signaled that the demonstration is communicative (Király et al., 2013; Butler et al., 2015; Hernik & Csibra, 2015). Clearly, marking a behavior as communicative changes its interpretation, but not in terms of possible object-directed intentions. Rather, it announces the presence of belief-directed intentions that inherently fall outside the scope of object-directed mentalizing.

Third, non-human primates share our ability to interpret behavior in terms of object-directed mental states (Hare, Call, & Tomasello, 2001; Krupenye, Kano, Hirata, Call, & Tomasello, 2016; Whiten, McGuigan, Marshall-Pescini, & Hopper, 2009). This means that if object-directed intentions and mentalizing were sufficient for generating and interpreting communicative demonstrations, we would expect to see them in non-human primates. However, there is limited evidence that non-human primates engage in such behaviors (Tomasello, 2010).

To understand communicative demonstrations thus requires characterizing both object-directed *and* belief-directed intentions as well as how observers reason about these distinct types of mental states from observed behavior. By formalizing these planning and inference processes, our aim is to explain a number of core features of teaching with and learning from demonstrations.

A Rational-Pragmatic Account of Communicative Demonstrations

Although our goal is to understand demonstration and observational learning, like others (Tomasello, Carpenter, Call, Behne, & Moll, 2005; Csibra & Gergely, 2009; Tomasello, 2010; Sperber & Wilson, 1986) our approach is inspired by past treatments of a different medium of human communication: language.

Current models of language agree that humans typically interpret statements in part by asking, “what is the speaker trying to communicate to me?”—i.e., by using belief-directed mentalizing. Ironic statements provide a vivid illustration of this principle. Suppose a person says: ”Look at this rain! I’m sure glad I didn’t bring my umbrella.” If a listener attempts to interpret this statement literally she will impute strange beliefs or desires to the speaker (e.g., he must not know what umbrellas do, or he must like getting wet). On the other hand, if the listener can distinguish the semantics of the speaker’s utterance from a representation of what the speaker is trying to *communicate* (Grice, 1957), then she has the ability to entertain non-literal interpretations (e.g., he is trying to communicate that he is disappointed he forgot his umbrella). Such non-literal interpretations are, of course, the essence of irony (Jorgensen, 1996).

We propose that communicative demonstrations rely on high-level formal principles that are similar to those in language pragmatics and pedagogy (Frank & Goodman, 2012; Shafto, Goodman, & Griffiths, 2014). Like the mime who takes the “wrong” cello-playing actions to successfully convey the idea of playing the cello, an ironic speaker communicates by saying things that are literally different, even contradictory to, what they actually mean. In particular, we mathematically characterize the generation and interpretation of communicative demonstrations in terms of demonstrators and observers recursively reasoning about one another’s behavior in a *cognitive hierarchy* (Camerer, Ho, & Chong, 2004).

Crucially, however, learning from demonstration typically involves learning something like a skill: How to perform a series of actions in order to accomplish some goal. Thus, the relevant domain of learning is *value-guided decision-making* (Dayan & Niv, 2008; Newell & Simon, 1972). There has been considerable interest in understanding how we can infer hidden decision variables (for instance, beliefs, desires and plans) from action observation, both in computer science (Abbeel & Ng, 2004) and psychology (Baker, Saxe, & Tenenbaum, 2009; Baker, Jara-Ettinger, Saxe, & Tenenbaum, 2017). Thus, in order to model learning from demonstration, we hybridize cognitive hierarchy models with *inverse planning*. This approach allows us to understand communicative demonstrations in explicit information- and decision-theoretic terms, which not only provides enhanced theoretical power but also the possibility of unifying a range of disparate theories and findings.

Translating ideas from the language setting to the decision-making setting, however, introduces new challenges and questions. For instance, in language, words clearly have a grounded semantics that is well-established, but meanings can depart from semantics via pragmatics (Grice, 1957; Goodman & Lassiter, 2015). How can non-linguistic demonstrations have their own grounded “semantics” that play an analogous role in communicative demonstration? Additionally, a communicative demonstrator must trade off communicative goals (e.g., “I want her to know how to make a good pizza”) against non-communicative goals and environmental constraints (e.g., “In the end, I’d also like to have a good pizza”). How does a demonstrator manage these potentially competing demands?

In addition, the analogy to language aside, communicative demonstration raises profound issues in its own right. For an observer, how does interpreting a demonstration as communicative differ from interpreting it as non-communicative? From the broader perspective of cultural transmission, what is the functional benefit of interpreting actions as communicative for imitation and learning? We aim to address these issues by formalizing a model of communicative intention, and then investigating the nature of learning at different levels of cognitive hierarchy.

Theoretical Contributions. Our account comprises three substantive claims. First, communicative demonstrations involve the interaction of two types of demonstrator intentions: object-directed intentions that are about outcomes in the environment, and belief-directed intentions that are about the mental state of an observer. A communicative demonstrator acts to jointly satisfy these two types of intentions, while a communicative observer interprets behavior on the assumption that both are operative. Second, in order to accomplish belief-directed goals, a demonstrator must be able to affect an observer's belief state through her actions. We argue that demonstrators actively leverage an observer's capacity for first-order theory of mind (Premack & Woodruff, 1978; Wellman, 1992; Malle & Knobe, 1997; Tamir & Thornton, 2018) in order to influence her beliefs and convey mental structure. Third, an expanded capacity for theory of mind then enables an observer to interpret behaviors not only in terms of object-directed intentions, but also in light of belief-directed intentions.

Formally, we model the decision-making and inferential processes during communicative demonstrations as an actor engaging in rational planning (Newell & Simon, 1972; Puterman, 1994) in an observer's belief space, and an observer engaging in inverse planning (Baker et al., 2017, 2009) in that space as well. This formulation illuminates the interaction of value-guided decision-making (i.e., rational planning) and cognitive hierarchy (i.e., observer interpretation by inverse planning). In particular, we can predict the trade-offs people will make when taking actions to satisfy both object-directed goals and belief-directed goals, which leads to exaggerated or altered behaviors, like the mime using the "wrong" actions to better convey cello-playing. We characterize and identify several such effects: *Targeted variability*, in which a demonstrator takes actions that pick out features that best distinguish an intention or mental structure; *informative inefficiencies*, in which actions with high signaling value are taken at a cost; and *expectation signaling*, in which the interpreted intention behind an action provides information beyond that of the actual outcome of an action.

At the same time, we can model how an observer's inferences and learning depend greatly on whether the actor has signaled a communicative episode, or whether instead the observer does not assume that the actor is demonstrating (e.g., because she does not know that she is observed). Specifically, once an observer can attribute actions not only to object-directed goals but also to belief-directed goals, that observer can draw richer inferences about the mental structure reflected in a demonstration. We characterize and identify two such effects: *Inferential amplification* and *crowding*, where the relative diagnostic values of demonstrations become magnified as a result of interpreting behavior as belief-directed; as well as *deviation* and *adherence attribution*, in which non-communicative, intentional behavior acts as a baseline for interpreting belief-directed intentions.

Empirical Evidence. To test the predictions and explanatory scope of our model, we report the results of several new experiments, and we also illustrate how the results of three previously published developmental experiments follow uniquely from higher-order mentalizing within our framework. In the new experiments, people interact with GridWorld environments that are simple but allow for a range of expressive behaviors. Participants in the role of the demonstrator take actions in the tasks in order to gain bonuses as well as communicate aspects of the task to an observing participant. Our model of planning in observer belief space successfully predicts a number of qualitative features of human communicative behaviors. Moreover, we are able to recover interpretable high-level parameters (e.g., how motivated people are to communicate) by fitting our model to low-level responses (e.g., the sequences of actions taken in the task).

Additionally, our model can shed new light onto previous developmental findings on imitation in communicative contexts. We analyze three studies reported in the literature: A study by Király et al. (2013) examining how infants differentially imitate subgoals based on communicative context; a study by Butler and Markman (2012) examining how generic causal representations are learned in accidental, intentional, and communicative contexts; and a series of studies by Hernik and Csibra (2015) investigating how tool functions are learned in communicative contexts. Our rational-pragmatic model can explain how observers integrate information about environmental context, communicative context, and demonstrator actions, and we find that it can successfully account for the various qualitative findings previously reported. This not only provides additional support for our account, but also an opportunity to unify multiple empirical findings in a single theoretical framework as well as explore predictions to test in future studies.

Summary. The article is structured as follows. We start by providing an intuition for the formulation and predictions of our framework, and then describe our computational model in detail. We then report two sets of experiments using a GridWorld problem solving paradigm in which participants demonstrated or inferred rewards of a task or relevant causal structure in a task. The subsequent section focuses on how our model explains previous developmental findings on infant observers of communicative demonstrations. Finally, we end by discussing how our modeling framework extends existing computational models of language pragmatics and pedagogy, how it connects to high-fidelity communication, over-imitation, costly signaling, and other proximal mechanisms of human sociality, and what it suggests for future research directions.

Model

Overview

We treat communicative demonstrations as emerging from the interaction of a number of well-studied cognitive processes. In particular, we combine ideas from rational planning (Newell & Simon, 1972; Puterman, 1994), inverse planning (Dennett, 1987; Baker et al., 2009) and recursive social reasoning (Sperber & Wilson, 1986; Camerer et al., 2004; Shafto et al., 2014; Frank & Goodman, 2012). In inverse planning, an observer interprets another's observable actions as having been generated through a process of rational planning given certain desires and beliefs. For example, observing someone reach for a pencil can be explained by them knowing there is a pencil nearby, wanting a pencil, and them being able to plan a sequence of arm movements that will successfully obtain a pencil. Although these inferences require a sophisticated model of unobservable desires, beliefs, intentions, and their relation to actions, there is considerable evidence that adults and infants have this capacity to some degree (Malle, 2008; Gergely & Csibra, 2003).

Theory of mind has been used to examine how people interpret behavior in light of beliefs and desires about the environment—that is, object-directed mentalizing. For example, previous work has looked at how observers reason about agents that are navigating to a location (Gergely et al., 1995), act based on imperfect information about the world (Baker, Saxe, & Tenenbaum, 2011), or take actions that have physical consequences for other agents (Heider & Simmel, 1944; Hamlin, Ullman, Tenenbaum, Goodman, & Baker, 2013). These settings involve inference about object-directed mental states in the sense that the observer reasons about demonstrator representations whose content are objective, physical states of the world. At the same time, the observer's capacity for object-directed mentalizing also provides a unique opportunity for the demonstrator: Should she desire, she can strategically select actions that cause the observer to make specific inferences about her object-directed mental states. In other words, she can represent and assign rewards to the observer's belief state.

Our approach builds on prior models of belief-directed decision-making. In particular, computational accounts of language pragmatics (Frank & Goodman, 2012; Goodman & Frank, 2016) and pedagogy (Shafto et al., 2014; Rafferty, Brunskill, Griffiths, & Shafto, 2016) have examined how utterances or examples are chosen in order to communicate a meaning or concept to another person. We apply and extend these ideas in three ways. First, we propose that object-directed mentalizing provides the foundation that supports the generation and interpretation of communicative demonstrations. That is, an observer's object-level theory of mind serves as a "hook" by which a demonstrator can actively convey beliefs she has about the world. Second, we explicitly formulate communicative demonstrations as a planning problem over object-directed and belief-directed representations. In other words, we treat communication with an observer as another goal that can be traded off with other rewards and costs in the environment. Third, in doing this, we can then model the interpretation of communicative demonstrations as mentalizing about object-directed and belief-directed demonstrator representations.

By grounding the interpretation of actions in object-directed intentions and treating communicative demonstrations as a single planning problem, our model can account for a number of empirical phenomena. For example, because communicative demonstrations must realize both object-directed and belief-directed goals, we can describe how people trade off these objectives when taking sequences of actions. These trade-offs manifest themselves in behaviors that seek to convey information about mental structure while maximizing object-level rewards or even at the expense of object-level rewards (e.g., taking inefficient, repetitive, or potentially costly actions). Similarly, we can explain how an observer attributes actions to a demonstrator's joint object-directed and belief-directed mental states in order to glean information about what the demonstrator knows.

In summary, we provide a unified cognitive account of communicative demonstrations in terms of belief-directed planning and inverse planning. Our model describes how factors such as environment context, communicative goals, rational planning, and observer inferences will interact in a range of settings and contexts.

Object-Directed Mentalizing as Inverse Planning

From a computational perspective, intention recognition involves representing and reasoning about hidden mental states or hidden properties of the environment based on observable behaviors. This, in turn, relies on having a representation or generative model of intentional behavior (Baker et al., 2009). Thus, we start by first reviewing a model of intentional behavior and then return to the question of inference.

Intentional Behavior as Planning. Following previous work, we formalize object-directed intentional behavior and activity as a solution to an object-level Markov Decision Process (MDP) (Jara-Ettinger et al., 2015; Puterman, 1994). An object-level MDP, M, is a tuple $\langle \mathcal{S}, \mathcal{A}, T, R \rangle$: a set of object-level states $\mathcal{S}$; a set of actions $\mathcal{A}$; a transition function that maps object-level states and actions to distributions over next states, $T : \mathcal{S} \times \mathcal{A} \to P(\mathcal{S})$; and a reward function that maps state/action/next-state transitions to scalar rewards, $R : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$. For example, in the GridWorld task we use in Experiments 1 and 2, object-level states can be considered locations, actions can be movements in the cardinal directions, transitions are the causal rules that dictate where one lands after taking an action from a state, and a reward function corresponds to points received for being on certain types of squares.

An MDP is a description of the rewards and dynamics that define a task faced by an agent, and the "solution" to a task is how the agent acts and responds to the environment. A plan is a

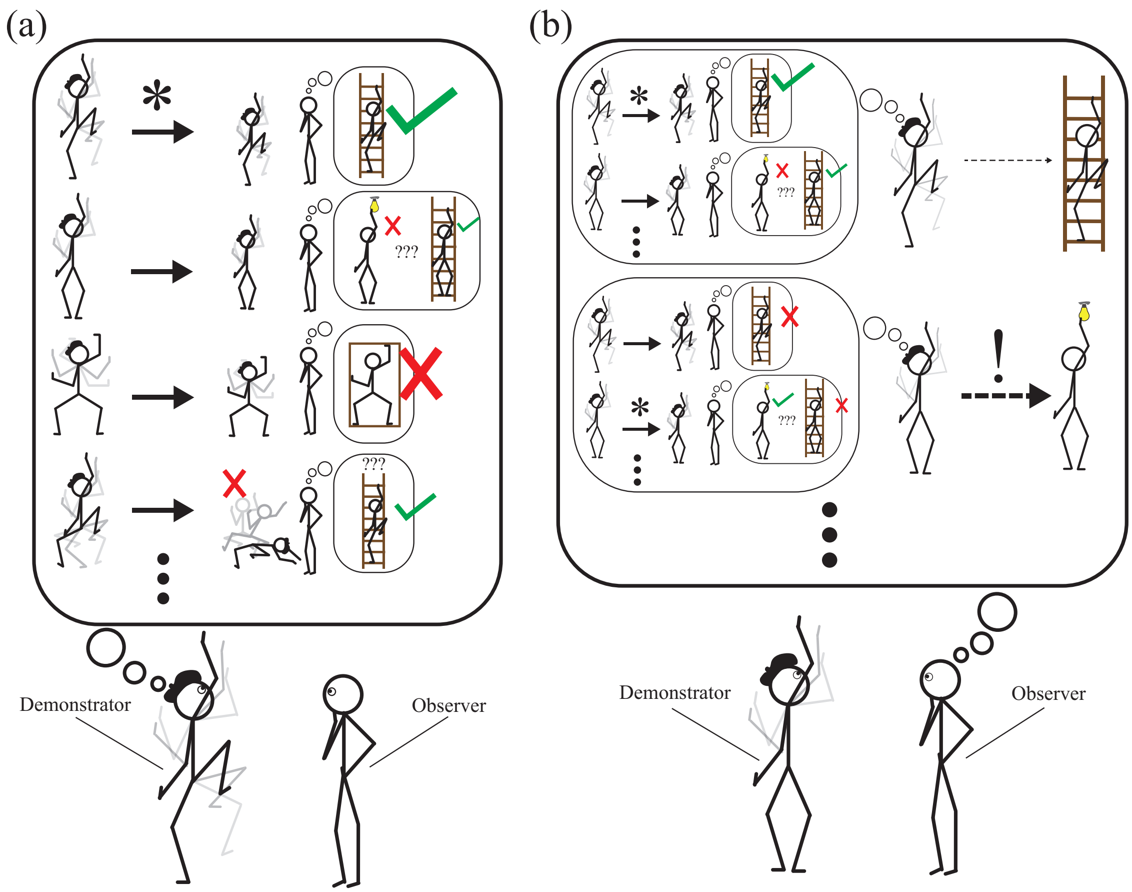

Figure 1. Rational-pragmatic model of communicative demonstrations. (a) A demonstrator wants to convey "climbing a ladder" to an observer. Solid arrows go from possible demonstrator action sequences to object-belief outcomes. She decides what sequence of actions to take based on whether it accomplishes belief-directed goals (e.g., that the observer infers "climbing a ladder") as well as object-directed goals (e.g., not falling on the floor). Importantly, the demonstrator leverages the observer's capacity for theory of mind and action interpretation in order to influence her beliefs. For example, the top action sequence successfully conveys climbing without falling over and is therefore optimal (marked with "*"); the second sequence somewhat conveys climbing but is ambiguous and less optimal; the third sequence conveys being in a box rather than climbing; the fourth sequence somewhat conveys climbing but results in the demonstrator falling over. Although we only display four demonstrator action sequences, many more are possible. (b) An observer reasoning about a demonstrator's communicative intentions can result in *inferential amplification*. Dotted arrows go from possible communicative intentions and actions to inferences about what is being conveyed. The demonstrator takes an action that is potentially ambiguous between climbing a ladder and installing a lightbulb, but since the observer reasons about what the demonstrator would have done with different communicative intentions, she can more strongly infer that the demonstrator wants to communicate "installing a lightbulb" (indicated with "!").

sequence of actions that an agent intends to take from a starting position, while a policy is a "conditional plan" that dictates what action should be taken given the agent finds herself in a particular state. Formally, a stochastic policy for an object-level MDP M maps object-level states to distributions over actions, $\pi : \mathcal{S} \to P(\mathcal{A})$. A policy is better or worse depending on how well it tends to solve the task. On a particular "run" of a policy from an initial state s_0, the interaction of an agent's policy and the environment dynamics will produce a trajectory of states and actions, $(s_0, a_0, s_1, ..., s_t, a_t, s_{t+1}, ...)$. This trajectory also produces a sequence of rewards, $(r_0, r_1, ..., r_t, ...)$, that can be combined either by considering a discount rate, $\gamma \in [0, 1)$, or a finite horizon into the future. A discount rate close to 0 means that temporally distant rewards are scaled to be less valuable than temporally close rewards, while a high discount rate means they are treated similarly. A finite horizon indicates how many timesteps into the future an agent plans.

Thus, from a particular state s, we can calculate what the expected, cumulative, discounted rewards would be by following π, otherwise known as the *value* of a state given a policy, $V^\pi(s) = \mathbb{E}_\pi\Big[r_0 + \gamma r_1 + ... + \gamma^t r_t + ... \mid s_0 = s\Big]$. An optimal policy yields the highest value for every state, and we will denote the optimal policy for a given MDP M_i as π_i, and corresponding optimal value function as V_i. The optimal state-action value function is the value of taking an action and then following the optimal policy from the next state, $Q_i(s, a) = R_i(s, a) + \gamma\mathbb{E}\Big[V_i(s')\Big]$.

To account for deviations from perfect optimality, we modify the optimal policy using a standard soft-maximization temperature $\tau \geq 0$ and random choice parameter, $\varepsilon \in [0, 1]$ (Luce, 1959; Nassar & Frank, 2016; Collins & Frank, 2018). The softmax temperature captures "value-related" randomness, where $\tau = 0$ corresponds to always choosing the highest-valued action, a higher temperature entails more value-related randomness, and $\tau \to \infty$ means actions are chosen uniformly randomly. The ε parameter corresponds to randomness that is not attributable to value, where $\varepsilon = 0$ is no non-value related randomness. How these two sources of sub-optimality combine are described in Appendix A.

Inverse Planning. By seeing another person's object-directed actions, an observer can use their understanding of intentional behavior to infer the goals, beliefs, or expectations of the demonstrator. That is, the demonstrator's actions provide information about how they view a task because of the links between mental representations, intentionality, and actions in the world provided by theory of mind.

Formally, an observer has a belief about what task M_i the demonstrator is solving, $b(M_i)$. As the demonstrator acts in the world, the observer updates this belief using an "inverted" model of intentional planning (Baker et al., 2009) according to Bayes Rule:

$$\begin{aligned} b'(M_i \mid s, a, s') &\propto P(a \mid s, M_i)P(s' \mid s, a, M_i)P(M_i) \\ &= \underbrace{\pi_i(a \mid s)}_{\text{Policy}}\underbrace{T_i(s' \mid s, a)}_{\text{Transitions}}\underbrace{b(M_i)}_{\text{Prior}}. \end{aligned} \tag{1}$$

The observer's inference combines two sources of information aside from their prior belief, $b(M_i)$. First, the demonstrator's policy, π_i, represents how they would do the task if it were M_i and so reflects their goals as well as their expectations about the structure of the environment. Second, the transition function, T_i, allows them to reason about how actual outcomes in the environment obtain. In the next section, we will show how a demonstrator who also has communicative goals can plan over this model and exploit these two information channels.

Belief-Directed Planning and Inverse Planning

Inverse planning models capture how people can use theory of mind to reason about a demonstrator's object-directed intentions. But what if the demonstrator also has goals with respect to the *belief state* of the observer? In that case, along with reasoning about features of objects in the environment, they should reason about the effects of their actions on the observer's beliefs. That is, communicative demonstrations involve planning in observer belief space. Critically, in our account, an observer's object-directed mentalizing provides a way for the demonstrator to "hook into" their inferential processes and for a communicative observer to reason about belief-directed intentions.

Observer Belief Space. Just as actions can be object-directed and used to influence object-level states, they can also be belief-directed and used to influence another's mental states. We can refine the previous object-level MDP to include beliefs by defining an Observer Belief Markov Decision Process (OBMDP) that a demonstrator plans over to produce communicative actions. Given a true object-level MDP, an OBMDP has the same actions available. The states in an OBMDP are extended to include not only the object-level states in the environment, but also the observer's beliefs, $\mathcal{B}$, which results in a joint object-belief state space: $\mathcal{S}^{\mathcal{B}} = \mathcal{S} \times \mathcal{B}$.

In general, the belief state space, $\mathcal{B}$, will be made up of infinitely nested observer beliefs that are induced by the observer reasoning about the demonstrator reasoning about the observer, etc. (Zettlemoyer, Milch, & Kaelbling, 2009). However, we do not deal with any of these structures directly. Rather, we assume that the demonstrator is only concerned with particular features of the belief state that represent different possible belief transitions and rewards[1]. We are mainly concerned with a case where the demonstrator cares about the observer's belief about the object-level MDP. As in the previous section on inverse planning, we will use the notation $b(X)$ to mean "the observer's beliefs about X." For example, $b(R_i)$ is the observer's belief in the reward function, while $b(M_i)$ would be their belief in all the variables of the object-level MDP M_i (e.g., including the transition function). Later, we will discuss nested observer beliefs in more detail.

Transitions in Observer Belief Space. A communicative demonstrator not only represents an observer's belief state, but also reasons about how object-level actions and transitions result in belief space transitions. But this raises a question: What belief-state transition model do people use? Our proposal is that *the belief transition model is grounded in an observer's capacity for object-directed mentalizing.*

Formally, we define *belief update functions* that output updated observer beliefs based on observed object-level transitions and previous observer beliefs, $\mathbb{B} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \times \mathcal{B} \rightarrow \mathcal{B}$. The "level-1" observer belief update function, $\mathbb{B}^{(1)}$, assumes the observer believes that the demonstrator does not have belief-directed intentions and only has object-directed intentions. That is, the observer reasons only about the demonstrator solving object-level MDPs, according to Equation 1. Thus, given a true object-level transition function, T_i, the joint level-1 object-belief transition function resulting from their composition is:

$$T_i^{\mathcal{B}1}(s', b' \mid s, b, a) = \underbrace{T_i(s' \mid s, a)}_{\text{Object Transition}} \; \underbrace{\delta\Big(\mathbb{B}^{(1)}(s, a, s', b), b'\Big)}_{\text{Belief Transition}}, \tag{2}$$

[1] In some cases, the demonstrator may have uncertainty about the observer's belief state, which we do not explicitly model here. However, we note that in the case of uncertainty without interaction, an optimal demonstrator would just select actions based on the probabilistically weighted (i.e., expected) belief state and transitions, which can be reduced to an equivalent (certain) OBMDP. Future work will need to explore the interplay of mutual uncertainty and interaction (i.e., see Rafferty et al. (2016)).

where $\delta(b_0, b_1)$ is a function taking on 1 or 0 depending on if b_0 and b_1 are the same, which results in deterministic transitions in belief space (but not necessarily in the object-level state space).

Rewards in Observer Belief Space. Given a representation of observer belief states and transitions within belief space, a demonstrator can have motivations that take those belief states as their content—i.e., belief-directed goals. This corresponds to rewards that are a function of belief states. Moreover, in the pedagogical settings considered here, two assumptions are made about the relationship between the object-level MDP and belief-directed rewards: (1) The *principle of knowledge*—that the demonstrator has true beliefs about the object-level MDP—, and (2) the *principle of honesty*—that the demonstrator wants to convey what they believe to be the structure of the world. Thus, the demonstrator is interested in changes in the observer's belief with respect to the true MDP: $\Delta b(M_i) = b'(M_i) - b(M_i)$. This then becomes a factor in the demonstrator's rewards:

$$R_i^{\mathcal{B}}(s, b, a, s', b') = \underbrace{R_i(s, a, s')}_{\text{Object Reward}} + \underbrace{\kappa\Big(\Delta b(M_i)\Big)}_{\text{Belief Reward}}, \tag{3}$$

where $\kappa \geq 0$ controls the degree to which a demonstrator is motivated to communicate relative to object-level rewards.

Formulating the reward function as in Equation 3 accomplishes several goals. First, it clarifies the assumptions relating the object-level MDP to belief-directed goals (i.e., the principles of knowledge and honesty). Second, it makes transparent how belief-directed motivations involve wanting to place the observer in a particular belief state. Finally, it explicitly treats object-directed and belief-directed motivations as separate factors that combine to direct the demonstrator's selection of actions. This enables us to model tradeoffs between object-directed goals and belief-directed goals, which inevitably arise in generating communicative demonstrations.

Belief-directed Planning. We have defined the notion of a joint object-belief state space, $\mathcal{S}^{\mathcal{B}}$; transitions in that space, $T_i^{\mathcal{B}1}$; and rewards over such states and transitions, $R_i^{\mathcal{B}}$. Along with the action space of the true object-level MDP, $\mathcal{A}$ we now have all the elements to characterize an OBMDP: $M_i^{\mathcal{B}1} = \langle \mathcal{S}^{\mathcal{B}}, \mathcal{A}, T_i^{\mathcal{B}1}, R_i^{\mathcal{B}} \rangle$.

By casting the generation of communicative demonstrations as a planning problem, we can apply the same analytical tools used in the non-communicative planning case. Specifically, an OBMDP, $M_i^{\mathcal{B}1}$, with a true object-level MDP, M_i, has an optimal policy, $\pi_i^{\mathcal{B}1} : \mathcal{S}^{\mathcal{B}} \to P(\mathcal{A})$, that maximizes the expected combined object-directed and belief-directed rewards under a model of transitions in the joint object-belief state space. Given an initial object and belief state, a state-action trajectory can be generated by repeatedly querying the policy. Moreover, analogous softmax-ε policies can be defined that capture deviations from perfect optimality.

Inverse Belief-Directed Planning. The level-1 belief transition model, $\mathbb{B}^{(1)}$, assumes that the observer is reasoning solely about object-directed intentions in order to update their beliefs. However, an observer could also be reasoning about the belief-directed intentions of the demonstrator. That is, they could be inverting a model of planning in observer belief space and trying to infer what the belief-directed intentions of the demonstrator are. Additionally, recall that in a pedagogical setting, it is assumed that the demonstrator is knowledgable and has honest belief-directed goals. This entails a one-to-one correspondence from OBMDPs to MDPs, which means that beliefs about the demonstrator's belief-directed intentions are equivalent to those about the object-level MDP.

We can define higher-level belief update functions that reason not only about object-directed goals in an object-level MDP, but also the belief-directed goals in an OBMDP. These belief update functions ultimately "ground out" in level-1 belief updates, but can be of arbitrary height. For example, a "level-2" belief update function, $\mathbb{B}^{(2)}$, reasons about possible *communicative policies* that are solutions to OBMDPs with level-1 update functions, e.g., $\pi_i^{\mathcal{B}1}$. This becomes formally analogous to inverse planning as described by Equation 1, but where policies and transitions are conditioned on the object-level state (s) and level-1 belief state (b):

$$b^{2\prime}(M_i \mid s, a, s', b, b') \propto \pi_i^{\mathcal{B}1}(a \mid s, b) T_i^{\mathcal{B}1}(s', b' \mid s, b, a) b^2(M_i). \tag{4}$$

The level-2 observer belief, b^2, corresponds to reasoning explicitly about belief-directed intentions, and it is worth emphasizing that this is distinct from the nested belief state of a level-1 observer, b. In reasoning about what a demonstrator with communicative goals would want a level-1 observer to infer, however, the level-2 observer can draw additional inferences about the demonstrator's beliefs about the structure of the world.

In theory, one can have communicative policies that solve OBMDPs with belief transition functions involving arbitrary nesting (e.g., $\pi_i^{\mathcal{B}2}, \pi_i^{\mathcal{B}3}, \pi_i^{\mathcal{B}4}, ...$). However, in the settings considered here, in which the principles of knowledge and honesty are assumed and what is being communicated is information about the object-level MDP, the behavior of higher-level communicative policies is qualitatively equivalent to the first two levels.

Summary. The framework presented here introduces several key ideas. First, communicative demonstrations depend on a representation of transitions in an observer's belief space. These transitions derive from an observer's capacity to reason about others' object-directed intentions and representations—their theory of mind (Figure 1a). Second, a knowledgeable and honest demonstrator's motivations are modeled as rewards dependent on the observer's belief in the true task. These belief-directed rewards can then be combined with other sources of rewards, such as those determined by the object-level task. Finally, communicative demonstrations result from planning over joint object-belief transitions to maximize joint object-belief rewards. Since these demonstrations result from planning, an observer can engage in inverse planning over belief-directed representations to infer the demonstrator's communicative intentions and, by extension, their object-directed representations (Figure 1b).

Qualitative Predictions of Model

Because we formalize mechanisms for generating and interpreting communicative demonstrations in fully-defined model of relationships among the environment context, the demonstrator's communicative goals, the observer's inference processes, and the demonstrator's actions, in principle we can make quantitative predictions. Specifically, given the initial beliefs of an observer and an environmental context, we can derive what types of actions a demonstrator will take and how those will update the observer's beliefs. Many experimental settings of interest, however, lack a sufficiently precise and comprehensive measurements to make use of such quantitative predictions—for instance, studies of how young children learn from demonstration.

Crucially, then, alongside the precise quantitative predictions made possible by a formal model, our account makes a number of interesting and distinctive qualitative predictions. Here, we discuss some of the qualitative predictions that our model makes for demonstrator and observer behavior and judgments. In the remainder of the article we show strong alignment between these predictions and qualitative patterns of human behavior.

Demonstrator Predictions. Our model posits that communicative demonstrators engage in belief-directed planning. Thus, they should tend to take actions that provide strong and unambiguous evidence for whatever fact they are trying to convey. In a word, the actions they choose should be diagnostic. This stands in contrast to a demonstrator who simply wants to accomplish the task itself—i.e., to maximize object-directed rewards. She has no incentive to act in a diagnostic manner; her goal is instead object-level efficiency. More precisely, then, our model predicts that in the case where there are multiple equally efficient ways to realize object-directed goals, a communicative demonstrator will engage in *targeted variability*: actions that emphasize the current intention and distinguish it from other intentions. In doing so, they help the observer pick out the particular features of the world relevant to their intentional behavior.

Our account also predicts how people will actively trade off non-communicative and communicative goals. That is, they may take actions that accomplish their communicative goals at the expense of object-level rewards. An example of these *informative inefficiencies* is when a demonstrator performs a highly diagnostic action repeatedly in order to provide additional evidence for the particular object-directed representation they want to convey. From the perspective of only maximizing object-directed rewards, repeating the same behavior is puzzling, but when planning also incorporates belief-directed rewards, such behaviors are sensible. Another example that exemplifies actively trading off different rewards is *expectation signaling*, which occurs when a demonstrator's actions convey that they expect a desirable outcome to occur, even if in fact a less desirable outcome happens to occur. Such behaviors are particularly interesting because they highlight how communicative demonstrators leverage an observer's specific capacity to reason about how intentions are determined by their general causal expectations, which can be distinct from what actually occurs at for a particular outcome. For example, the general expectation that starting the ignition turns on one's car can be reflected even in a particular instance where starting the ignition fails to do so.

Targeted variability, informative inefficiencies, and expectation signaling are a few of the qualitative phenomena predicted by our model. Additionally, because our account focuses on communicative intentions as an element of planning, it predicts that communicative demonstrators will actively seek out opportunities to engage in such behaviors, even if they are costly.

Observer Predictions. An observer who is cued into a demonstrator's communicative intent wants to infer the content of that intent and, by extension, the true structure of the environment. That is, they are reasoning jointly about object- and belief-level planning representations, or how a person would act if they were not just optimally doing a task, but also if they were actively conveying something. This can result in the phenomenon of *inferential amplification* on the part of the observer: If the action taken without belief-directed goals were even mildly diagnostic, then the same action taken with belief-directed goals can become extremely diagnostic. Put another way, since the communicative demonstrator's intent is to push the observer's belief in a certain direction, an observer who is cued into this intent can extrapolate where the demonstrator ultimately wants their belief to go. This can occur even if the non-communicative belief update and initial belief in a hypothesis is small, resulting in large differences in the communicative and non-communicative posterior belief distributions. Moreover, the amplification of one set of hypotheses can result in *inferential crowding* of others, in which alternatives are less likely because they do not account for behavior at the level of belief-directed goals.

An additional feature of the observer model is *deviation attribution*. Because the communicative demonstrator still cares about object-level rewards, their behavior will generally reflect this and be interpreted as such. However, whereas for a non-communicative observer, actions inconsistent with the optimal object-level policy are simply noise, for a communicative observer, such actions acquire meaning by being explainable in terms of communicative goals. Put another way, inferences about object-level intentions serve as a baseline against which communicative intentions can be made salient. As a result, when actions deviate from this baseline, they can be attributed to the communicative intent of the demonstrator.

The converse of deviation attribution is *adherence attribution*, which occurs when actions are perfectly consistent with a baseline behavior and exhibit no deviation. In adherence attribution, a communicative observer may draw the inference that because the demonstrator did

(a)

	Exp 1: Reward Structure	Exp 2: Causal Structure
Targeted Variability	✓	
Informative Inefficiencies	✓	✓
Expectation Signaling		✓

(b)

	Király, Csibra, & Gergely, 2013	Butler & Markman, 2012	Hernik & Csibra, 2015
Inferential Amplification/Crowding	✓	✓	✓
Deviation/Adherence Attribution	✓	✓	

Figure 2. Qualitative predictions that appear in new and previously reported experiments. (a) Predictions when generating communicative demonstrations versus non-communicative behavior. *Targeted variability* occurs when a demonstrator picks out features that distinguish the current intention from others. *Informative inefficiencies* occur when a demonstrator goes out of their way to take a highly diagnostic action (potentially repeatedly) in order to provide additional evidence for what they want to convey. *Expectation signaling* occurs when a demonstrator's intended outcome provides information above and beyond the actual outcome. (b) Predictions when interpreting communicative demonstrations versus non-communicative demonstrations. *Inferential amplification/crowding* refers to how actions that are mildly diagnostic/anti-diagnostic of a mental state in a non-communicative context become highly diagnostic/anti-diagnostic in a communicative one. *Deviation/adherence attribution* refers to how deviations from a non-communicative baseline of intentional behavior can be attributed to belief-directed intentions in a communicative context.

not deviate when they could have, they must want to communicate precisely the intention that is consistent with their actions.

Inferential amplification/crowding and deviation/adherence attribution highlight phenomena that emerge from object-directed and belief-directed mentalizing. Additionally, they can interact with one another and other aspects of a communicative observer's inferential processes. For instance, inferences made by attributing deviances to communicative goals can be amplified. Later, we apply our model and these predicted phenomena to explain several findings in the developmental literature.

Generating and Interpreting Communicative Demonstrations

Our account of communicative demonstrations as belief-directed planning predicts particular types of demonstrator behaviors and observer inferences. To test these predictions, we developed a paradigm in which participants would play either the role of the demonstrator who take actions with the goal of conveying the structure of an environment, or the role of the observer who reports their inferences.

In particular, we focused on two elements of a task that a demonstration might communicate: reward structure and causal structure. From the perspective of value-guided decision-making, an agent's representation of these two variables plays a critical role in determining how they can behave adaptively in the environment. In order to maximize rewards in their environment, an agent must know what features of that environment are rewarding. For example, to stay healthy, it is useful to know that firm, red apples are safe to eat, while mushy, blue apples are dangerous to eat. How demonstrators convey information about rewards is the focus of Experiments 1a and 1b.

Meanwhile, in order to effectively plan to obtain rewards, one must be able to model the environmental consequences of one's actions. For example, pressing buttons on remote controls can turn on and control televisions, which is a causal property that is useful to know if one wants to be entertained. Experiments 2a and 2b focus on how demonstrators convey and observers infer such representations of causal structure in a goal-directed context.

Throughout both studies, we use non-communicative behavior as a baseline against which to compare communicative demonstrations. That is, we compare participants who are doing and showing the task to those who are only doing the task. This serves several purposes. First, having a non-communicative comparison for problem-solving behavior allows us to explicitly test if certain behavioral signatures that characterize phenomena like targeted variability, informative inefficiency, and expectation signaling are more likely to occur in a communicative context. Second, because communicative demonstrations are inherently the product of both object-directed and belief-directed mental states, we can test if the additional variance explained by planning in observer belief space is specific to the demonstration being communicative. Relatedly, the comparison allows us to determine if the specific fitted model parameters are interpretable in light of the two conditions. For instance, if people are truly engaging in belief-directed planning, we would find demonstrator parameters reflecting a greater sensitivity to changes in an observer's beliefs as well as a motivation to place observers in a particular belief state.

Finally, we note that both studies use a GridWorld paradigm consisting of a set of contiguous "tiles" with different features and available actions that allow participants to move an agent around the world. Although the ground state space of our GridWorld domains are finite and relatively small, they allow participants to engage in an arbitrarily large number of distinct action sequences within a trial. Since our model of belief-directed planning makes specific predictions about how environmental constraints and demonstrator goals result in sequences of actions, the open-endedness of these tasks provides an opportunity to test the generality of our framework.

In short, the experiments reported here test how representations involved in value-guided decision-making are conveyed and interpreted through communicative demonstrations. We use a variety of behavioral measures and model-fitting approaches to compare the predictions of our model of planning in observer belief space with human data in open-ended GridWorld domains. Across both experiments, we found that our framework can successfully capture the richness of human communicative demonstrations.

Experiment 1: Communicating Reward Structure by Demonstration

Rewards often depend on features of an environment. For example, pointy objects are often dangerous to touch while red apples are often delicious to eat. In general, people often use feature-based representations for generalization (Austerweil & Griffiths, 2013), and feature-based reward functions are used extensively in reinforcement learning (e.g., Abbeel & Ng, 2004). As a result, a demonstrator's knowledge about reward-relevant features are reflected in their behavior, which means they can use their behavior to convey information about reward-relevant features. In Experiment 1, participants were given the feature-based reward task shown in Figure 3 and either played the role of a demonstrator (Experiment 1a) or observer (Experiment 1b).

We find that belief-directed planning can explain key differences in non-communicative and communicative settings. In particular, using both behavioral and model-based analyses, we find robust evidence that people engage in a combination of targeted variability and informative inefficiencies that reflect belief-directed planning. Our model-based analyses also show how our belief-directed planning model can recover psychologically interpretable high-level parameters corresponding to the constructs of our framework. Moreover, we find that communicative demonstrators provide demonstrations that observers can draw more accurate and confident inferences from, consistent with our account.

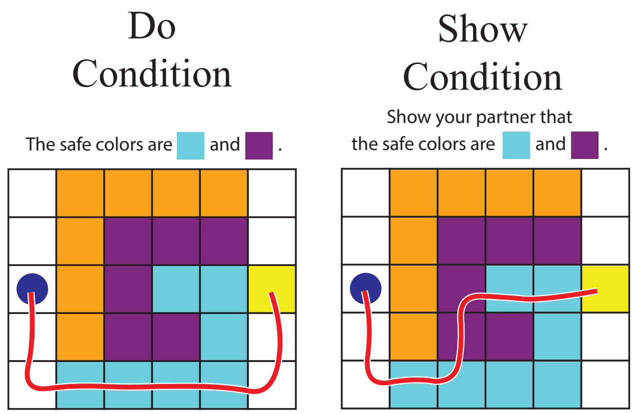

Figure 3. Experiment 1 - Participants were either placed in a condition where they were simply told the reward function (left) or also told to show the reward function to a partner (right). The red lines are representative examples of behavior in the two conditions.

Method

Task. We used the feature-based reward GridWorld task shown in Figure 3. Each participant can control the blue circle, which can be moved to any adjacent tile in one of the four cardinal directions (north, south, east, west) on each time step. Each trial, the agent starts in the same location on the middle left. Each tile has a single color: white, orange, purple, blue, or yellow. On each trial, the yellow tile is the "goal", meaning that whenever an agent enters it, they always receive +10 points and that trial ends. It is always in the same location. Additionally, across all trials, the white tiles always reward 0 points. On each trial, the orange, purple, and blue tiles could each be *safe* or *dangerous*, meaning that their reward values could either be 0 or -2 points respectively. The set of reward functions formed by all combinations of safe or dangerous yields a space of eight possible distinct reward functions.

Task Model. We model each trial as its own MDP, M_i, with the same set of states, actions, transition dynamics, and discount rate, but a different reward function, R_i. To make the role of reward-based features explicit, we define a state feature function, ϕ, that maps each $s \in \mathcal{S}$ to a binary 5-dimensional vector where each entry corresponds to one of the colors (in order: white, yellow, orange, purple, or blue). The reward function is determined by a reward weight vector θ_i. For example, when purple and blue are dangerous, $\theta_i = [0, 10, 0, -2, -2]$. The reward for ending up in a blue state s' after taking action a in state s is determined by the feature function applied to s', $\phi(s') = [0, 0, 0, 0, 1]$, and the reward weight vector, yielding $R_i(s, a, s') = \theta_i^\top \phi(s')$. The observer starts with a uniform distribution over eight possible values of θ_i, corresponding to uncertainty about whether each of the orange, purple, and blue rewards are zero or -2.

Procedure. Sixty participants recruited from Amazon Mechanical Turk performed the feature-based reward teaching task; two were excluded due to missing data due to recording error, leaving a total of 58 participants for analysis. They were given a base pay of $1.00 and received a bonus based on points received across the whole experiment with each point worth +/- 2 cents. The experiment was organized into a training phase and test phase. The training phase was designed to familiarize participants with the domain by having them do a series of trials that alternated between learning a reward function and then applying it on a subsequent trial. On the learning trials, they were *not* told the underlying reward function (i.e. which colors were safe/dangerous), but received immediate feedback on how many points were won or lost when stepping on tiles. On the applying trials immediately following each learning trial, they were given a new grid configuration that required knowledge of tile color type, and applied what they just learned about the tiles without receiving feedback. They all played 8 pairs of learning and applying rounds corresponding to the 8 possible assignments of “safe” and “dangerous” to the orange, purple, and blue tiles. The order of the reward functions was randomized between participants.

Following the first phase, participants were split into two conditions, Do and Show. Do participants were told which colors were safe and won points for performing the task. Show participants were also told which colors were safe and also won or lost points based on the ground reward function. They were additionally told that their behavior would be shown to another person, that this person would apply what was learned from observing their behavior to a new grid, and that the points won by their partner would be added to their bonus. When bonuses were calculated, participants each received what they would have had their partner done as well as possible. Participants did not receive feedback on the reward for each action, although this could be easily inferred from the information provided. Procedures were approved by Brown University’s Research Protection Office (protocol #1505001248, title: “Exploring human and machine decision-making in multi-agent environments”).

Results

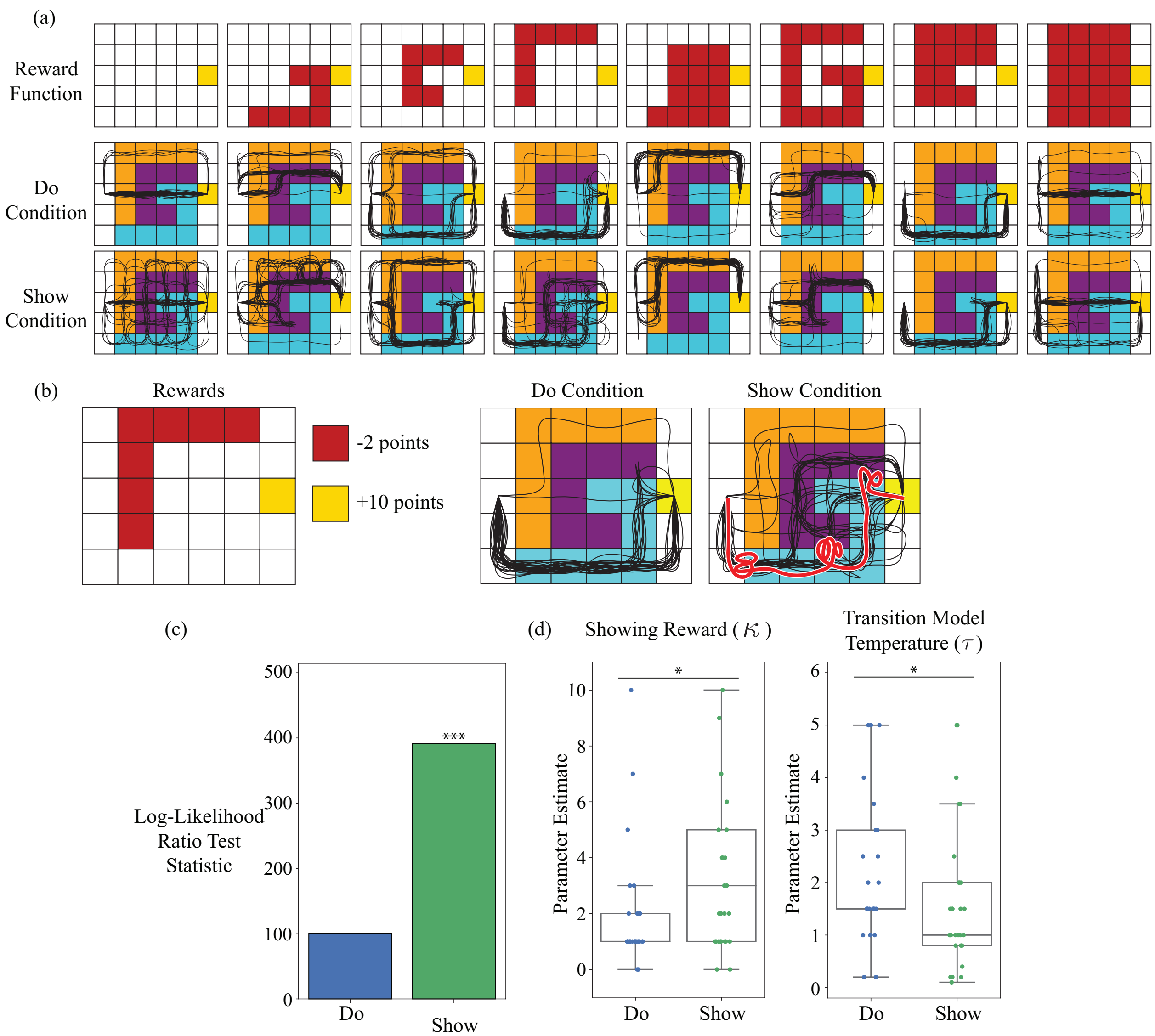

Figure 4. Experiment 1 results. (a) Participant demonstrations by trial. Top row: Ground rewards of the task. Red tiles are -2 points, white tiles are 0 points, and yellow tiles are worth $+10$ points and end the trial. Middle row: Do participant trajectories with visible tile colors; Bottom row: Show participant trajectories. (b) An exaggerated demonstration (red line) in the Show condition. Behaviors such as this are predicted by the planning in observer belief space model. (c) Log-likelihood ratio test statistic values for participant models by condition (29 participants per condition). For each participant, the planning in observer belief space model has 4 additional parameters associated with changes in an observer's beliefs (see text), so the test statistic is approximately a χ^2-distribution with 116 degrees of freedom (Wilks, 1938). (d) Fitted participant parameters by condition. Note that for the showing reward weight, outliers ($\kappa > 10$) are not plotted but were included in all analyses. Tests are Wilcoxon rank-sum tests. * : $p < .05$. Panel (a) adapted from Ho, Littman, MacGlashan, Cushman, and Austerweil (2016).

Participant demonstrations, visualized in Figure 4a, matched the qualitative and quantitative predictions of the model. Do participants largely took efficient routes to reach the goal state, whereas Show participants took paths that signaled feature reward values. We analyzed the data from two perspectives. First, we tested whether behaviors characteristic of targeted variability and informative inefficiency differed between the two conditions. Specifically, we analyzed steps spent on the orange, purple, and blue tiles versus other tiles, steps spent on safe color tiles versus the white tiles, steps that revisited tiles, the overall variety of tiles visited, and the length of trajectories. Showing participants scored higher on all of these measures. Second, we performed a model-based analysis where we fit the object-directed planning model and belief-directed planning model to participant data. We find that belief-directed planning captures the particular variability in the Show condition.

Behavioral Analyses. Our behavioral analyses focused on two features that characterize belief-directed planning. First, Show demonstrations will have more *targeted variability* than non-communicative demonstrations. That is, a greater variety of tile-types will be visited, but specifically in a manner that ensures disambiguation. Second, Show demonstrations will have *informative inefficiencies*. For instance, participants might revisit states that have high diagnostic value and demonstrations will be more temporally extended than needed to successfully complete the task.

To assess targeted variability, we first examined the amount of time spent on the uncertain colors (orange, purple, blue) versus the certain colors (white, yellow). Overall, participants visit uncertain colors more per trial in Show than Do (Do: Mean=0.62, Median=0.67, S.E.=0.01; Show: Mean=0.69, Median=0.71, S.E.=0.01). We also fit a mixed-effects logistic regression model with whether an action was taken from a unknown reward color tile (orange, purple, or blue tile) or not as the binary outcome variable. Intercepts across participants and reward functions were treated as random effects, and condition (Do/Show) as a fixed effect. Variance in participant ($SD = 0.28$, $\chi^2(1) = 58.38$, $p < .0001$) and reward function intercepts ($SD = 0.31$, $\chi^2(1) = 75.67$, $p < .0001$) were significant. Variance in the trial number slopes was not significant for participant ($\chi^2(2) = 0.47$, $p = .79$) or reward function ($\chi^2(2) = 0.05$, $p = .97$), and so neither were included in the final model. The fixed effect of condition was significant ($\beta = 0.40$, $SE = 0.10$, $Z = 4.04$,

$p < .0001$ [Wald Z test]). The same model, but with safe-color tile (as opposed to any of the unknown colors) as the binary outcome variable showed the same effect by condition ($\beta = 0.53$, $SE = 0.12$, $Z = 4.57$, $p < .0001$ [Wald Z test]). In short, participants in Show were more likely to visit unknown color tiles, particularly the ones that were safe on a trial, as compared to those in Do.

We can examine variability within a demonstration itself by calculating the entropy (all values use log-base of 2) of the frequency distribution over unknown color-tile visits. This was generally greater in Show than in Do (Do: Mean=0.57, Median=0.57, S.E.=0.02; Show: Mean=0.72, Median=0.81, S.E.=0.04). To analyze this measure, we fit a mixed-effects linear regression with it as the outcome variable. We found significant variance in random intercepts for participants ($SD = 0.13$, $\chi^2(1) = 21.72$, $p < .0001$) and reward functions ($SD = 0.45$, $\chi^2(1) = 352.28$, $p < .0001$). By-participant and by-reward function trial number slopes were not significant. The final model had condition as a significant fixed effect ($\beta = 0.16$, $SE = 0.05$, $t(57.04) = 3.30$, $p < .01$ [Satterthwaite's approximation]), confirming that those in the showing condition additionally visited a greater variety of color tiles.

Redundancy was compared by looking at the number of steps in a demonstration and whether locations were revisited within a demonstration. Show participants had longer demonstrations (Do: Mean=7.86, Median=9.00, S.E.=0.12; Show: Mean=10.99, Median=9.00, S.E.=0.38). We fit a mixed-effects linear regression with random intercepts for participants ($SD = 2.78$, $\chi^2(1) = 197.36$, $p < .0001$) and reward functions ($SD = 1.08$, $\chi^2(1) = 10.29$, $p < .001$) and without random slopes for trial number. In the final model, condition was significant ($\beta = 3.13$, $SE = 0.78$, $t(57.75) = 3.99$, $p < .001$ [Satterthwaite's approximation]).

Finally, we analyzed location-revisitation (i.e. revisiting a specific location on the grid). Show participants revisited states more (Do: Mean=0.10, Median=0.00, S.E.=0.03; Show: Mean=1.38, Median=0.00, S.E.=0.23). We performed a mixed-effects logistic regression over responses with whether a state was revisited as the binary outcome variable. Variance in random intercepts for participants ($SD = 1.47$, $\chi^2(1) = 536.91$, $p < .0001$) and reward function ($SD = 0.67$, $\chi^2(1) = 67.74$, $p < .0001$) was significant. Only the variance in random trial number slopes for participants was significant ($SD = 0.26$, $\chi^2(2) = 25.57$, $p < .0001$) and included in the model. For the final model, condition was set as a fixed effect and was significant ($\beta = 2.09$, $SE = 0.54$, $Z = 3.90$, $p < .0001$ [Wald Z test]), indicating that when generating communicative demonstrations, people were more likely to revisit specific locations in the task.

Model-based Analysis. To determine how well our model holistically accounts for behavior, we fit the belief-directed planning model to individual participants. This involves identifying the most likely observer belief MDP that a demonstrator is solving. We then consider a space of models parameterized by seven values: The discount rate and ε-softmax values of nested object-directed planners $(\gamma, \tau, \varepsilon)$; the showing discount rate and ε-softmax values of the observer belief MDP $(\gamma^{\mathcal{B}}, \tau^{\mathcal{B}}, \varepsilon^{\mathcal{B}})$; and the belief-directed reward weight (κ). Since transitions in the observer belief MDP are determined by how well an action distinguishes one possible MDP from another, the nested object-directed parameters $(\gamma, \tau, \varepsilon)$ control the "degree of information" that a demonstrator assumes an observer will infer. Meanwhile, the parameters involved in belief-directed planning $(\gamma^{\mathcal{B}}, \tau^{\mathcal{B}}, \varepsilon^{\mathcal{B}}, \kappa)$ reflect a communicative demonstrator's general motivation and strategy for conveying information. Details of of how parameters were estimated from data are reported in Appendix C, while model implementation details are reported in Appendix B.

In our account, communicative demonstrations result from joint-planning over object-directed and belief-directed goals and representations. This means that for some parameterizations of an observer belief MDP, e.g. $\tau \to \infty$, $\varepsilon = 1$, and $\kappa = 0$, an observer belief MDP degenerates into the original ground MDP. Thus, to assess the validity of our observer belief MDP as a model of showing, we conducted likelihood-ratio tests with $\tau = 1000$, $\varepsilon = 1$, and $\kappa = 0$ as the null model (i.e. an MDP in only the true ground task determines transitions or rewards). This makes the total difference in degrees of freedom four per model. We first analyzed whether the planning in observer belief space model provides a significantly better account of human data by analyzing aggregate behavior by condition (with parameters fit per participant). As shown in Figure 4c, for Do, the null ground model was not rejected ($\chi^2(116) = 100.78$, $p > .85$), while for Show, it was ($\chi^2(116) = 391.60$, $p < .001$), indicating that the planning in observer-belief space model only provides a better explanation of behavior for the Show condition. Examining tests at the individual level, more individual models in Show (21 of 29) rejected the null ground model than in Do (3 of 29) at the .05 significance level (i.e. participants for whom $\chi^2(4) > 9.49$, $p < .05$;

Fisher's exact test, $p < .001$).

Given fits to individual participants, we can also assess whether specific parameters in belief-directed planning model meaningfully capture participant motivations and representations (Table 1). As shown in Figure 4d, we show that this is the case. We examined three variables: the belief-directed reward weight parameter κ and the nested object-directed planner ε-softmax parameters, τ and ε. Between the two behavioral conditions, we find that the estimated κ is higher for Show indicating that people were more motivated to convey to an observer what task they were performing (Wilcoxon Signed-Ranks test: $Z = -2.11$, $p < .05$). Additionally, we find that estimated τ is *lower* in Show as compared to Do, which indicates that people are representing their actions as having a greater capacity to signal their intentions (Wilcoxon Signed-Ranks test: $Z = 1.98$, $p < .05$). Moreover, we find that the two parameter estimates to be correlated in Show but not Do, confirming the coupling between the representational and motivational dimensions of communicative demonstrations (Do: $r = .27$, $p = 0.16$; Show: $r = .50$, $p < .01$). For estimated ε, we did not find a significant difference (Wilcoxon Signed-Ranks test: $Z = 1.38$, $p = .17$).

In short, the model-based analyses in this section illustrate how the planning in observer belief space model can uniquely account for global properties of participant behavior when communicating feature-based rewards by demonstration. Our model can also recover interpretable parameter values corresponding to theoretical constructs from human data.

Experiment 1b: Learning Reward Structure from Demonstrations

To examine whether the demonstrations generated by participants in Show were more effective at conveying reward structure, we recruited a separate set of participants to learn the reward values of each color by observing trajectories from the two conditions in Experiment 1a. Additionally, to assess the relative importance of an observer interpreting demonstrations as intentionally communicative, we manipulated the interpretation of these demonstrations by telling them that the demonstrator was intentionally communicating or not. Overall, we found a large positive effect on observer accuracy and confidence when given demonstrations from the Show condition, and a small positive effect based on observer interpretation (regardless of whether the demonstrations were from the Do or Show condition).

	Do	Show
γ	0.96 (0.01)	0.93 (0.01)
ε	0.12 (0.02)	0.09 (0.01)
τ	2.20 (0.25)	1.64 (0.25)
κ	2.55 (0.74)	5.31 (1.35)
$\gamma^{\mathcal{B}}$	0.93 (0.01)	0.93 (0.01)
$\varepsilon^{\mathcal{B}}$	0.04 (0.01)	0.05 (0.01)
$\tau^{\mathcal{B}}$	0.15 (0.03)	0.22 (0.04)

Table 1

Experiment 1 model parameter estimates. Means and standard errors across participants ($n = 29$ for each condition).

Materials and Procedure. The stimuli were the state/action/next-state sequences produced by participants in Experiment 1a. These were generated from the eight critical trials from the 29 participants the Do/Show demonstrator conditions, for a total of 464 demonstrations. Each participant was told they would observe a single demonstration from a partner. They were also assigned to a Communicative or Non-Communicative interpretation condition. The instructions were the same except participants in the Communicative condition were also told that their partner "knows that you are watching and is trying to show you which colors are safe and dangerous". Next, they were shown a page with the animated demonstration and answered, for each of the three colors (orange, purple, and blue), whether they thought it was safe or dangerous and their confidence on a continuous scale (0 to 100). Each participant received a starting payment of 25¢ and won/lost 5¢ for each correct/incorrect answer (minimum payment was 10¢). Two MTurkers were assigned to each demonstration and observer instruction combination using psiTurk (Gureckis et al., 2016). Procedures were approved by University of Wisconsin-Madison ED/SBS IRB (Study #2017-0830, title: "Studying human and machine interactions").

Results. For both observer accuracy and observer confidence, we found main effects of demonstrator and observer conditions. To analyze accuracy, we used a repeated-measures logistic regression with correct/incorrect judgments as the outcome variable. Trial (reward function), demonstrator, and observer were used as random effects, and both sets of instructions were used as fixed main effects. The effect of whether the trajectories were from Do or Show (demonstrator instructions) was significant ($\beta = 0.40$, $SE = 0.11$, $Z = 3.64$, $p < .001$ [Wald Z test]) as was the effect of the Communicative/Non-Communicative manipulation (observer instructions) ($\beta = 0.13$, $SE = 0.07$, $Z = 1.97$, $p < .05$ [Wald Z test]). Demonstrator Show instructions had a larger effect size, corresponding to an increase in observer accuracy by 1.5 times, as compared to observer Communicative instructions, which corresponds to an increase in observer accuracy by 1.14 times.

Confidence judgments were analyzed with a mixed-effects linear regression model. Reported confidence was the outcome variable; trial, demonstrator, and observer were random effects; and demonstrator and observer instructions were fixed effects. Observers receiving Show demonstrations were more confident ($\beta = 3.34$, $SE = 0.93$, $t(57.20) = 3.59$, $p < 0.001$ [Satterthwaite's approximation]), as were those receiving Communicative instructions ($\beta = 3.57$, $SE = 0.87$, $t(1790.80) = 4.08$, $p < 0.0001$ [Satterthwaite's approximation]). Thus, the generation and interpretation of demonstrations as communicative increase both accuracy and confidence.

Discussion

The reward structure of an environment directly affects the object-directed intentions and plans of a demonstrator, which means an observer can infer the unobservable rewards guiding observed actions. Our account argues that communicative demonstrators will leverage the observer's capacity for object-directed mentalizing during belief-directed planning. Consistent with this claim, we find that participants in the Show condition engage in targeted variability, in which they will strategically modify their behavior to expose the specific range of safe state features. Moreover, people will engage in specific types of informative inefficiencies that are sub-optimal in the ground MDP but send strong signals about the unobservable reward structure of a trial. For example, participants take more steps in a trial or even repeat the exact same sequence of states and actions.

Additionally, the results of our model-based analyses illustrate how our account can explain the global structure of communicative demonstrations in terms of interpretable, high-level quantities. We formulate communicative demonstrations as a specific type of planning problem involving a true ground MDP, a model of observer inferences, and belief-directed motivations. This allows us to directly compare models and conditions with and without communicative components, and show the specific role of belief-directed planning during communicative demonstrations. Furthermore, when we fit our models to individual participants in the two conditions and estimate the contribution of different planning variables to measured behavior, we can recover interpretable parameter values. This illustrates the theoretical utility of treating communicative demonstrations as a form of planning over object-directed and belief-directed representations.

Finally, we find that Show demonstrations are more effective for conveying the unobservable reward structure of a task to another person, and that the observer's interpretation of a demonstration also affects their accuracy and confidence. These findings are consistent with our account and provide further support for communicative demonstrations as involving planning and inverse planning over observer belief representations.

Experiment 2: Communicating Causal Structure by Demonstration

Actions can convey not only reward structure but also the *causal structure* that a demonstrator knows about the world. But object-directed intentional actions alone often do not provide enough information to identify causal structure, particularly in the case of tool use (Gergely & Csibra, 2006). For example, imagine an observer learning how to use a can-opener from a demonstrator who has no belief-directed intent. The demonstrator would quickly align the cutting mechanism of the opener with the lip of the can, squeeze the opener's arms, puncture the can, and twist the outer handle to roll the cutting mechanism around the can. Even if all her actions were visible, it would be unclear what aspects of that particular action sequence are most important—e.g. an observer may not realize that before squeezing the handle, the gear just behind the cutter needs to be positioned to "bite into" the lip of the can from below, otherwise the puncturing mechanism will not have leverage. A demonstrator with a belief-directed goal would attempt to provide this additional information about the relevant causal mechanism by, say, exaggerating how the gears, cutter, and can lip must align before squeezing. Although this elaboration of the action sequence would be inefficient for accomplishing the goal of opening the can, it effectively conveys how and why a particular action sequence opens a can, knowledge that is crucial for the observer to generalize to other cans and can-openers.

Thus, Experiment 2 examines how people will modify their object-directed intentional behavior (e.g. opening a can) in order to convey information about deeper causal structure (e.g. how can-openers open cans). We used a different GridWorld task in which certain tiles have different possible stochastic causal outcomes when acted upon (Figure 5), which allows us to examine how people and our model can plan for different possible outcomes and even exploit them to accomplish belief-directed goals. Overall, we find that our model strongly predicts how people will act to convey causal structure through their actions. In particular, we find clear evidence that people engage in the various forms of targeted variability, informative inefficiencies, and expectation signaling predicted by our account.

Method

Task. We used the task shown in Figure 5a to test how people generate communicative demonstrations to convey causal structure. As before, participants control the blue circle and can move it in one of the four cardinal directions on each time step. Each trial, the agent starts in the same location on the bottom center of the grid and wants to reach the yellow goal tile (worth 50 points). Reaching the goal tile ends the trial, and each step taken in the trial costs one point. Additionally, there were two other types of color tiles on the grid. Dangerous tiles (red) always result in a loss of 10 points, while “jumper tiles” (green) are worth zero points themselves. Unlike any of the other tiles, when the agent steps off of a jumper tile, it will sometimes “jump” two steps in whatever direction was taken rather than only one. Jumper tiles can be either be “strong”, meaning that 3/4ths of the time the tile moved the agent two steps and 1/4th of the time moved it only one step, or they could be “weak”, in which the probabilities were reversed. When an agent “jumps” two tiles, they “skip” the intermediate tile and any of its (positive or negative) rewards. As a result, the value of actions from a particular jumper tile depends on both the layout of the dangerous and jumper tiles, as well as whether the jumper tiles are strong or weak. Importantly, jumper tiles are all of the same type within a given trial.

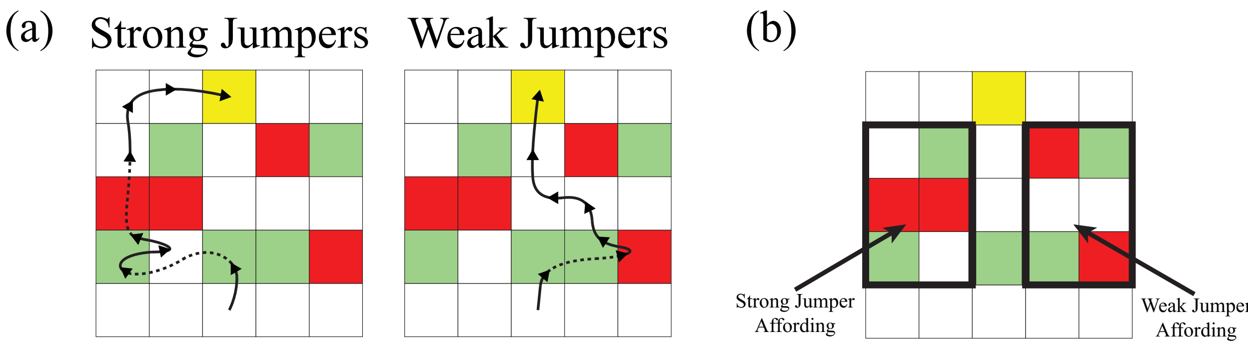

Figure 5. Experiment 3 paradigm. Each trial had a particular configuration of dangerous tiles (red) and jumper tiles (green). (a) Example trajectories depicting whether the jumper tiles are *strong* (the agent is usually moved two tiles after stepping off it) or *weak* (agent is only sometimes moved two tiles). Dotted line indicates a successful jump of two tiles. Within a trial, jumper tiles are either all strong or all weak. (b) Example of regions of grid with different affordances based on whether the jumper tiles are strong or weak.

Task Model. Similar to Experiment 1, we can model each trial as its own MDP, M_i, with the same set of states, actions, discount, and feature-based reward function, but with different feature-based transition functions, T_i. We define a state feature function, ϕ that maps each tile state $s \in \mathcal{S}$ to a 6-vector where the first four entries are binary and correspond to color (white, yellow, red, green), and the last two entries correspond to the x, y coordinates of the tile. The distribution over next states given the previous state and action are defined using transformations over the different features. For example, when the green tiles are strong jumpers, taking the action `up` from a green tile increments the value of the x feature by two with probability 3/4, and by one with probability 1/4 (assuming that the green tile is at least two tiles away from the top edge of the grid). On each trial, the observer starts with a uniform distribution over two transition functions corresponding to the green tiles being strong or weak.

Procedure. 80 Amazon Mechanical Turk participants (22 female, 58 male) took our study for payment. The overall design of this experiment was similar to that of Experiment 1 with a few modifications. Participants were trained on the basic experimental interface and interacted with a set of 16 exploration grids in which they had to figure out whether the jumper tiles were strong or weak. The grids were designed such that there was no way to repeatedly sample without risking entering a dangerous tile. After each of these exploration rounds, they had to answer whether they thought the jumper tiles on that trial were strong or weak and won or lost 50 points based on their answer. They were then split into two conditions: Do and Show. Forty-one participants were assigned to Show while 39 were assigned to Do. In Do, participants were always told whether the jumper tiles were strong or weak; in Show, they were also told this information but were additionally told that their behavior would be shown to a partner who would have to answer whether the jumper tiles were strong or weak. They would then win or lose 50 points based on their partner's answer.

Both conditions were given the same set of 8 grids twice. We designed the grids to have permutations of regions that were better to go through when the jumper tiles were weak (*weak affording*), strong (*strong affording*), or where it did not matter (Figure 5b). The availability of differentially affording regions between the start and goal state gives participants opportunities to use the jumper tiles in a variety of ways, allowing us to distinguish between doing and showing. Each grid was then presented where the jumper tiles were strong and weak, for a total of 16 distinct rounds per person. Procedures were approved by University of Wisconsin-Madison ED/SBS IRB (Study #2017-0830, title: "Studying human and machine interactions").

Results

Our model of belief-directed planning predicts that people will engage in targeted variability and informative inefficiencies that reflect demonstrator tradeoffs in ground rewards and communicative rewards. In the context of the jumping GridWorlds, this means people will go out of their way in order to take actions that are informative about the planning-related causal properties of the green tiles. For example, repeatedly jumping off of green tiles provides strong evidence of the underlying causal mechanism by giving observers the opportunity to directly observe the relevant statistics. An even stronger way to signal the expectation that a particular causal outcome is more likely is to take a *risky jump* where the outcome will have a large influence on what rewards an agent receives. For instance, if green jumper tiles are strong, an agent would use it to jump over a red tile, whereas if it were weak, they would not. As a result, an observer who observes the demonstrator attempt to use a green tile to jump over a red tile would infer that it is strong, even if they land in the red tile. This provides an opportunity for a communicative demonstrator to then use risky jumps to signal their knowledge about causal

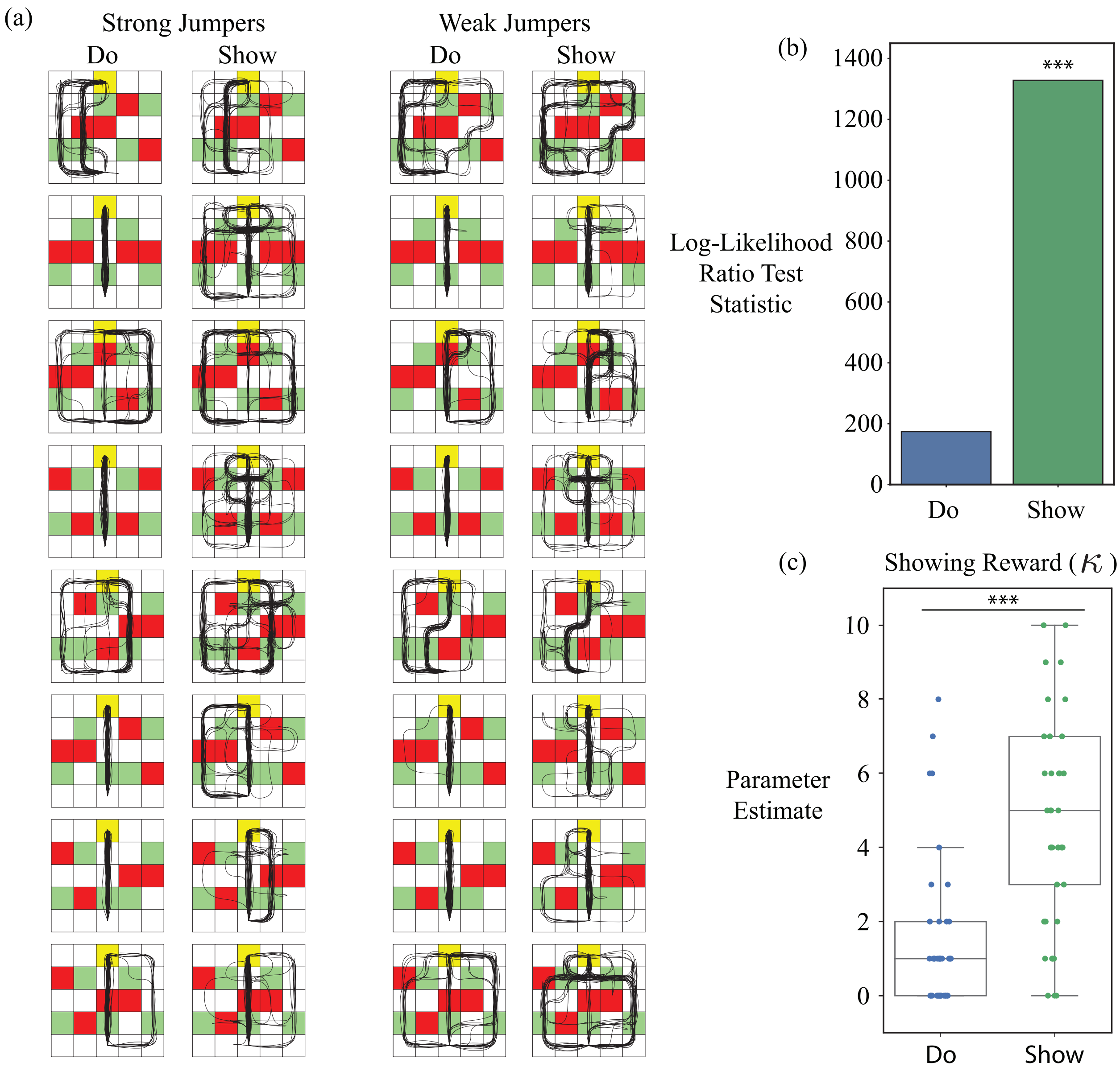

Figure 6. Experiment 3 results. (a) Participant trajectories by condition and trial. (b) Log-likelihood ratio test statistic values for by-participant fits collapsed over condition. (c) By-participant showing reward (κ) maximum likelihood parameter estimate by condition. Note that for the showing reward weight, outliers ($\kappa > 10$) are not plotted but were included in all analyses. Test is a Wilcoxon rank-sum test. $^*: p < .05$, $^{**}: p < .01$, $^{***}: p < .001$

structure. Further, it provides us an opportunity to disentangle demonstrations and learning based on expected versus observed outcomes.

Figure 6a plots trajectories from several experimental trials in the two conditions, and a visual inspection indicates the presence of certain qualitative differences predicted by the models.

To characterize these differences, we test for several behavioral signatures predicted by the model as well as perform analyses based on model-fits to the belief-directed planning model.

Behavioral Analyses. For our behavioral analyses, we focused on three measures: the length of demonstrations, the tendency to take any jump, and the tendency to take risky jumps. Longer demonstrations would be expected in Show, particularly if participants went out of their way in order to demonstrate causal structure (although this will not always be the case since optimally doing the task might require taking a longer route). Length of demonstrations was analyzed with a mixed-effects linear regression in which Do/Show condition was a fixed effect, and participant intercept, participant-trial number slope, and transition function (i.e. item) intercept as random effects. Consistent with our predictions, Show trajectories were longer than Do trajectories (Model Intercept: $\beta = 4.97$, $SE = 0.45$, $t(24.10) = 11.11$, $p < .0001$ [Satterthwaite's approximation]; Condition: $\beta = 1.85$, $SE = 0.31$, $t(77.62) = 6.05$, $p < .0001$ [Satterthwaite's approximation]).

Jumping between the two conditions also differed as predicted by the belief-directed planning model. General jump-taking was analyzed with a mixed-effects logistic regression with the same fixed and random effects as in the previous analysis. The estimated fixed effect of condition revealed that Show participants were 1.16 times more likely to take jumps than Do participants (Model Intercept: $\beta = -1.27$, $SE = 0.08$, $Z = -15.98$, $p < .001$ [Wald Z test]; Condition: $\beta = 0.15$, $SE = 0.06$, $Z = 2.44$, $p < .05$ [Wald Z test]). A stronger and potentially more efficient way to convey causal structure is to leverage the intention-reading capacities of an observer by taking *risky jumps*. This is because the risk associated with jumping has a strong signaling value due to the observer attributing mental states and expectations to the demonstrator. To analyze risky jumping, we fit a mixed-effects logistic model with the same fixed and random effects as the general jumping model, but where the predicted variable was whether a jump was a risky one or not (i.e. non-jump actions were not fit). The analysis revealed that Show participants were 2.94 times more likely to take a risky jump than non-risky jump than those in Do (Condition: $\beta = 1.07$, $SE = 0.23$, $Z = 4.47$, $p < .0001$ [Wald Z test]).

In short, our model of showing as belief-directed planning makes specific qualitative predictions about how demonstrators will act to convey the causal structure of an environment.

When communicatively demonstrating in the current setting, trajectories are longer, jumping attempts are more common, and, in particular, risky jumping attempts are more common. All three of these behavioral predictions are clearly borne out, providing converging support for the model.

Model-based Analyses. To determine how well the belief-directed planning model can account for the overall behaviors in the two conditions, we also performed a by-participant model-based analysis. Separate belief-directed planning models were fit to each participant in the two conditions, each of which had seven parameters. These were then compared with a nested null model in which $\tau^{\mathrm{Do}} \to \infty$, $\varepsilon^{\mathrm{Do}} = 1$, and $\kappa = 0$, which is mathematically equivalent to a standard planning model. The parameters tested are reported in Appendix C.

Likelihood ratio tests revealed that participant behavior from Do does not reject the pure object-directed planning model in favor of the belief-directed planning model ($\chi^2(156) = 174.27$, $p = .15$), whereas those from Show do reject the model ($\chi^2(164) = 1328.24$, $p < .0001$, Figure 6b). In other words, belief-directed planning captures how people modify their actions in order to show causal and affordance-related information about a task. Additionally, an individual-level analysis reveals that more individual hypothesis tests reject the null standard planning model in Show (33 out of 41) than in Do (8 out of 39) at the .05 significance level (i.e. participants for whom $\chi^2(4) > 9.49$, $p < .05$; Fisher's exact test: $p < .001$).

The participant-level fits also allow us to assess individual parameter estimates and whether they meaningfully reflect the psychological constructs posited by our model (Table 2). Figure 6c shows that this is the case for the showing reward weight parameter κ: A comparison between the two behavioral conditions shows that the estimated κ is higher for Show, which confirms that people were more motivated to convey to an observer what task they were performing ($Z = -5.04$, $p < .0001$ [Wilcoxon Signed-Ranks test]). Lower $\varepsilon^{\mathrm{Do}}$ and τ^{Do} would correspond to a more deterministic observer belief transition model. However, we did not find any detectable differences in $\varepsilon^{\mathrm{Do}}$ ($Z = 0.11$, $p = .91$ [Wilcoxon Signed-Ranks test]) or τ^{Do} ($Z = -0.98$, $p = .33$ [Wilcoxon Signed-Ranks test]) between the two conditions.

	Do	Show
γ	0.58 (0.05)	0.78 (0.05)
ε	0.25 (0.03)	0.24 (0.03)
τ	0.83 (0.27)	1.11 (0.28)
κ	1.49 (0.33)	5.68 (0.78)
$\gamma^{\mathcal{B}}$	0.84 (0.04)	0.76 (0.04)
$\varepsilon^{\mathcal{B}}$	0.03 (0.01)	0.18 (0.02)
$\tau^{\mathcal{B}}$	0.05 (0.01)	0.08 (0.01)

Table 2

Experiment 2a model parameter estimates. Means and standard errors across participants ($n_{Do} = 39$, $n_{Show} = 41$).

Learning Causal Structure from Demonstrations

Given that participants modify their behavior when generating communicative demonstrations, we can also ask whether these modifications are effective for conveying the underlying causal structure of a task. To test this, we recruited another set of participants to observe trajectories form the two conditions and provide judgments. As in Experiment 1, we also provided *Communicative* or *Non-Communicative* instructions to these participants to see if observer inferences would be affected by interpretation. We found that in this domain, demonstrations from Show were more effective at conveying the correct causal structure, but that there was no effect of observer interpretation.

Materials and Procedure. Three-hundred and twenty participants (150 female, 168 male, 2 other) were recruited via Amazon Mechanical Turk to participate in our study. Two participants were assigned to each of the 80 demonstrators from Experiment 2a in two conditions (*Communicative* and *Non-Communicative*) using psiTurk (Gureckis et al., 2016). After completing a consent form, participants were shown instructions explaining that they would watch their partner play a game, that their goal was to reach the yellow square on each round and win 50 points, that red squares caused them to lose 10 points, and that green tiles were jumper tiles. We emphasized that jumpers could be strong and "usually succeed" or weak and "usually fail".

Unlike in Experiment 1, where each participant saw a single trial, here they were told they would watch their partner play 16 trials of the game. We made clear that on each trial, the green tiles are either all strong or all weak, and they had to determine whether the jumpers on that round were strong or weak. They were given a starting bonus of 0.80 and won/lost 5¢ for each correct/incorrect answer and did not receive feedback during the experiment. Finally, in the *Communicative* condition, but not the *Non-Communicative* condition, they were also told “Your partner knows that you are watching and is trying to show you whether the jumpers on that round are strong or weak.” To ensure that participants understood all critical aspects of the task, we included comprehension questions in the instructions that needed to be answered correctly to proceed.

In the main part of the task, participants were shown the 16 demonstrations produced by one of the participants from Experiment 2 (in the same order). On each trial, participants could view a video of each demonstration as many times as they wanted (but at least once) and provided two judgments: whether the jumpers on that trial were strong or weak, and their confidence on a continuous slider ranging from “No Confidence” to “Extremely Confident”. After completing all 16 trials, participants were asked several post-task questions. Procedures were approved by University of Wisconsin-Madison ED/SBS IRB (Study #2017-0830, title: “Studying human and machine interactions”).

Results. We analyzed participants’ judgments of whether trial jumpers were strong and weak as well as their confidence using mixed-effects models. For jumper judgments, we coded responses as correct or incorrect based on whether they matched the true causal structure. A mixed-effects logistic regression was fit with correctness as the predictor variable; item (i.e. transition function and grid configuration), participant, and trial number intercepts as random effects; and demonstrator condition (Do/Show), observer condition (Communicative/Non-Communicative), and their interaction as fixed effects. We found a significant effect of demonstrator condition indicating that demonstrations produced with the intent to communicate increased correct judgments by 2.31 times ($\beta = 0.84$, $SE = 0.17$, $Z = 4.85$, $p < .0001$ [Wald Z test]). However, whether the observer perceived it as being communicative had no influence ($\beta = -0.05$, $SE = 0.17$, $Z = -0.30$, $p = .76$ [Wald Z test]) and was there also no interaction ($\beta = 0.08$, $SE = 0.24$, $Z = 0.36$, $p = .74$ [Wald Z test]).

For confidence judgments, we fit a mixed-effects linear model with confidence as the predictor variable; item, participant, and trial number intercepts as random effects; and observer condition, demonstrator condition, and their interaction as fixed effects. We found no effect of either demonstrator or observer condition.

To assess the influence of our manipulations on general observer attitudes, we analyzed several of the post-task question responses. Using a mixed-effects linear model for responses to "How easy or difficult was it to figure out whether jumper tiles were strong or weak?" (1-5), we found no influence of either condition (all $|t(311.00)| < 0.82$, $p > .41$ [Satterthwaite's approximation]). For the question "How helpful was your partner?" (1-5), we found that participants given demonstrations from Show gave higher responses ($\beta = 0.34$, $SE = 0.14$, $t(187.95) = 2.50$, $p < .05$ [Satterthwaite's approximation]). Similarly, a logistic-regression analysis on responses to "Do you think your partner was trying to be helpful?" showed more "Yes" responses to Show demonstrators ($\beta = 1.24$, $SE = 0.38$, $Z = 3.29$, $p < .01$ [Wald Z test]). In the same analysis, we found that participants in the *Non-Communicative* condition were less likely to say their partner was trying to be helpful ($\beta = 0.78$, $SE = 0.34$, $Z = 2.31$, $p < .05$ [Wald Z test]), confirming that the intended effect of our experimental instructions on observer interpretation persisted across the task.

Discussion

A person's representation of an environment's causal structure determines how they will act to realize object-directed goals and intentions, which means that such information can be inferred from as well as conveyed through their behavior. Our account predicts people will act based on planning in an observer's belief space when engaging in communicative demonstrations, which can involve leveraging opportunities to convey causal structure. In Experiment 2, we tested this prediction by having participants perform a Grid-World task in which a state feature (e.g. a tile being green) indicated that it had one of two causal properties (e.g. being a strong or weak jumper tile). Whether jumper tiles in a trial were strong or weak varied by trial and was unknown to the demonstrator but known to the observer. Model-fitting and behavioral analyses both revealed that the behavior in the Show condition was predicted by belief-directed planning, providing additional support for our account. Moreover, we found that observers were more successful at inferring causal structure from Show trajectories.

Additionally, the specific types of informative inefficiencies we found are revealing about how people convey causal structure during communicative demonstrations. For example, compared to participants in Do, those in Show tended to repeatedly take steps off of a jumper tile to convey the statistics of the causal system. This can be an effective way to indicate causal structure and corroborates results from Experiment 3 of Shafto et al. (2014) showing that people will optimally intervene in a system in order to teach causal concepts.

However, we also found that people will specifically engage in risky jumps that are detrimental to their expected object-directed rewards. Unlike repeatedly jumping off of a tile, which conveys causal structure by letting the observer see the frequencies of different outcomes, risky jumping relies on the observer's capacity to reason about object-directed intentions. Specifically, it requires that the observer be able to reason about what expectations the demonstrator has about possible outcomes (i.e. how far one is likely to jump) as well as how those integrate with rewards (i.e. the danger of landing on a red tile). Interestingly, risky jumping may also be a more efficient way to communicate causal structure since (1) even if the negative outcome occurs, expectations are conveyed, and (2) the expected outcome can be conveyed through a single action, unlike repeated non-risky jumping. Thus, the presence of risky jumping provides particularly strong evidence that communicative demonstrations specifically involve leveraging an observer's capacity for object-directed mentalizing.

Combined with the results of Experiment 1, the results of the present experiment illustrate the generality of our framework for modeling how different representations can be conveyed during communicative demonstrations. As in the first experiment, the belief-directed planning model can account for differences between behavior in the Do and Show conditions. Similarly, we find that people can infer the true causal structure better from Show trajectories. We note that unlike in Experiment 1, we did not find that the framing of the trajectories as communicative or not influenced observer inferences. This may be due to the fact that the space of possible causal structures was smaller (two versus eight) as well as that the particular structure of this task precludes observer inferences playing a large role. In the next several section, we analyze previous developmental studies in which infant observers were given demonstrations in communicative or non-communicative contexts. There we find cases in which observer interpretation has a large influence on inferences that are made.

Infant and Child Observer Studies

So far we have explored how demonstrators shape their behavior by planning in observer belief space. Next we explore how observers anticipate this form of planning, potentially interpreting demonstrations as communicative acts intended to modify their beliefs. When observers engage in this form of "inverse planning" (i.e., attempting to understand how the observer wishes to modify their beliefs) this often allow especially efficient and powerful forms of coordinated teaching and learning. This dimension of communication has been long recognized in work on language analyzing the pragmatic interpretation of utterances (Frank & Goodman, 2012; Kao, Wu, Bergen, & Goodman, 2014). The relationship between communicative intent and imitation has also been explored extensively in the developmental literature (Brugger et al., 2007; Southgate et al., 2009; Király et al., 2013; Hernik & Csibra, 2015; Buchsbaum et al., 2011; Butler et al., 2015; Sage & Baldwin, 2011; Hoehl et al., 2014). Having formalized this form of inverse planning in the context of communicative demonstrations, our goal is to compare the major qualitative predictions of the model against the empirical literature on children's learning from communicative demonstrations.

Why is inference especially powerful when a learner successfully anticipates that a demonstrator is planning in her belief space, and interprets her actions as such? After all, even if the communicative observer anticipates that the demonstrator has some belief-directed goal, she does not know for free which goal it is. Nonetheless, she can more strongly attribute behaviors to belief-directed goals and use that information to infer the underlying state of the world. That is, she can using inverse planning to infer joint object- and belief-directed mental states. Compared to only reasoning about object-directed intentions, this joint reasoning facilitates stronger inferences because the observer expects the demonstrator to take informative actions. This results in several specific phenomena that we explore in detail: *Inferential amplification*, where actions that are mildly diagnostic of an object-directed intention become highly diagnostic of a corresponding belief-directed intention; *inferential crowding*, in which less diagnostic alternatives are ruled out; *deviation attribution*, in which actions that would have been interpreted as noise under an object-directed interpretation become attributable to belief-directed goals; and *adherence attribution*, in which intentionally adhering to (not deviating from) a certain object-directed behavior becomes stronger evidence for its goal status.

Here, we revisit three studies with infants and young children in light of our account. In all of the studies, the infant or child played the role of the observer, and an experimenter played the role of the demonstrator. Additionally, all the studies had at least two conditions: One in which demonstrations were performed after having given a cue that something was being communicated to the observer (a *Communicative* condition), and one in which no cue preceded the demonstration (an *Intentional* condition). Each set of studies focused on a different type of mental structure that could be conveyed through demonstrations. Specifically, Király et al. (2013) focus on differential imitation of subgoals, Butler and Markman (2012) focus on learning generic causal properties, while Hernik and Csibra (2015) focus on inferring novel functional properties of tools. What unifies these findings is that all the mental structures being conveyed and inferred are value-guided decision-making representations, and so they will be reflected in the demonstrator's intentional behavior.

In fitting our model to this data our primary goal is to rigorously test the claim that the model is compatible with the basic qualitative dimensions of the data. We do not claim that our model is uniquely capable of explaining the findings, although we do remark on ways in which simpler "lesioned" versions of our model (e.g., one without planning, or inverse-planning, in belief space) would fail to capture the qualitative patterns. Throughout, we are required to make mathematically explicit a set of assumptions about environmental context, communicative context, and demonstrations in the original experiments. In formalizing these assumptions, we must make decisions about what details of a study are theoretically relevant. We have attempted to stay true to the original interpretations of the researchers while also balancing simplicity and maintaining the *a priori* commitments of our theoretical framework. Overall, we find that our account can formally describe key qualitative findings from these experiments.

Imitating Subgoals based on Communicative Demonstrations

Summary of Findings. Experiment 1 of Király et al. (2013) examined children's imitation of goal-directed behavior varying two factors: whether or not the demonstrator cued communicative intent, and whether or not the demonstrator's possible actions were notably constrained by environmental factors. In the modeling phase of their experiment, infants observed an experimenter sit down across from them and then bend over to use their head to touch a novel object, causing it to light up. This demonstration was performed in four different conditions using a 2×2 design. The first factor was whether the context was communicatively cued or not. In the Communicative conditions, the experimenter looked at the infant, called their name, and made sure they were paying attention before the demonstration. In the Intentional[2] conditions, the demonstrator did not interact with the infant, but waited until a signal was given from another experimenter that the infant was paying attention before performing the demonstration. The second factor manipulated whether the demonstrator's hands were occupied or free. In the Hands-Occupied conditions, the demonstrator was wearing a blanket and clutching it with their hands. In the Hands-Free conditions, they were wearing a blanket but their hands were placed on the table next to the novel object.

During the test phase children had the opportunity to interact with the novel object and thus, potentially, to imitate the actor. In the Communicative conditions, the demonstrator led the infant to the novel object and stayed in the room while they played with it. In the Intentional conditions, a different experimenter led the infant to the novel object and left the room. The main analysis examines whether the infants imitated the demonstrator (i.e. used their head to turn on the light, as opposed to using some other body part such as their hands) based on the two factors. Neither main effect was significant, but the interaction was significant. Specifically, in the Communicative condition, there was more imitation of the head action in the Hands-Free condition than the Hands-Occupied condition, whereas there was no detected difference in the Intentional condition (Figure 7).

Intuitively, a person who turns on a light with their head in the Hands-Occupied condition uses their head only because their hands are occupied, whereas a person who turns on a light with

[2]Király et al. (2013) use the term "Incidental" to describe this condition.

their head in the Hands-Free condition uses their head because it is necessary or preferable. Moreover, a person who communicatively demonstrates turning on a light with their head in the Hands-Free condition is choosing a highly diagnostic signal that head-use is important or preferable. Our model naturally captures these intuitive principles.

Model Formulation. We can formalize the experiment in our modeling framework[3]. Specifically, we begin by specifying the following two assumptions about the observer's prior: (1) The goal of touching the object to light it up is initially unknown to the observer (regardless of the means), and (2) a demonstrator is more likely to have using their hands as a subgoal than using their head. Formally, we assume that the observer is reasoning about a distribution over ground MDPs, $\mathcal{M}$, that differ in terms of their reward functions. We consider a set of three possible reward functions: $R_{\text{No-goal}}$, where touching the object is not a specific goal (all actions are 0 reward); $R_{\text{Use-Hands}}$, where touching the object is a goal and using one's hands is a subgoal (turning the light on is +10 reward and using one's hands is +10 reward); and $R_{\text{Use-Head}}$, where touching the object is a goal and using one's head is a subgoal (turning the light on is +10 reward and using one's head is +10 reward). The observer's prior belief is that touching the novel object is probably not a goal ($p_{\text{No-goal}} = 0.90$) but that if it is a goal, using one's hands is more likely than using one's head ($p_{\text{Hands}} = .08$, $p_{\text{Head}} = .02$). These values were chosen to reflect reasonable background assumptions in the task design, and we find qualitatively similar results for similar values of $p_{\text{No-goal}}$, p_{Hands}, and p_{Head}.

The environmental constraints in the Hands-Free and Hands-Occupied conditions can be modeled as two states with different available actions. In the HANDS FREE state, the demonstrator can take one of three actions: *Use Head*, *Use Hand*, or *Other*. The *Use Head* and *Use Hand* actions deterministically lead to the LIGHT ON state, while the *Other* action leads to a terminating state. Meanwhile, in the HANDS OCCUPIED state, the demonstrator can either take the action *Use Head* or *Other*, which both lead to the same successor state as in the HANDS FREE case.

We assume that the demonstrator models all select actions using an ε-greedy policy, with

[3]All models in this section were implemented using WebPPL (Goodman & Stuhlmüller, 2014) and code will be available online.

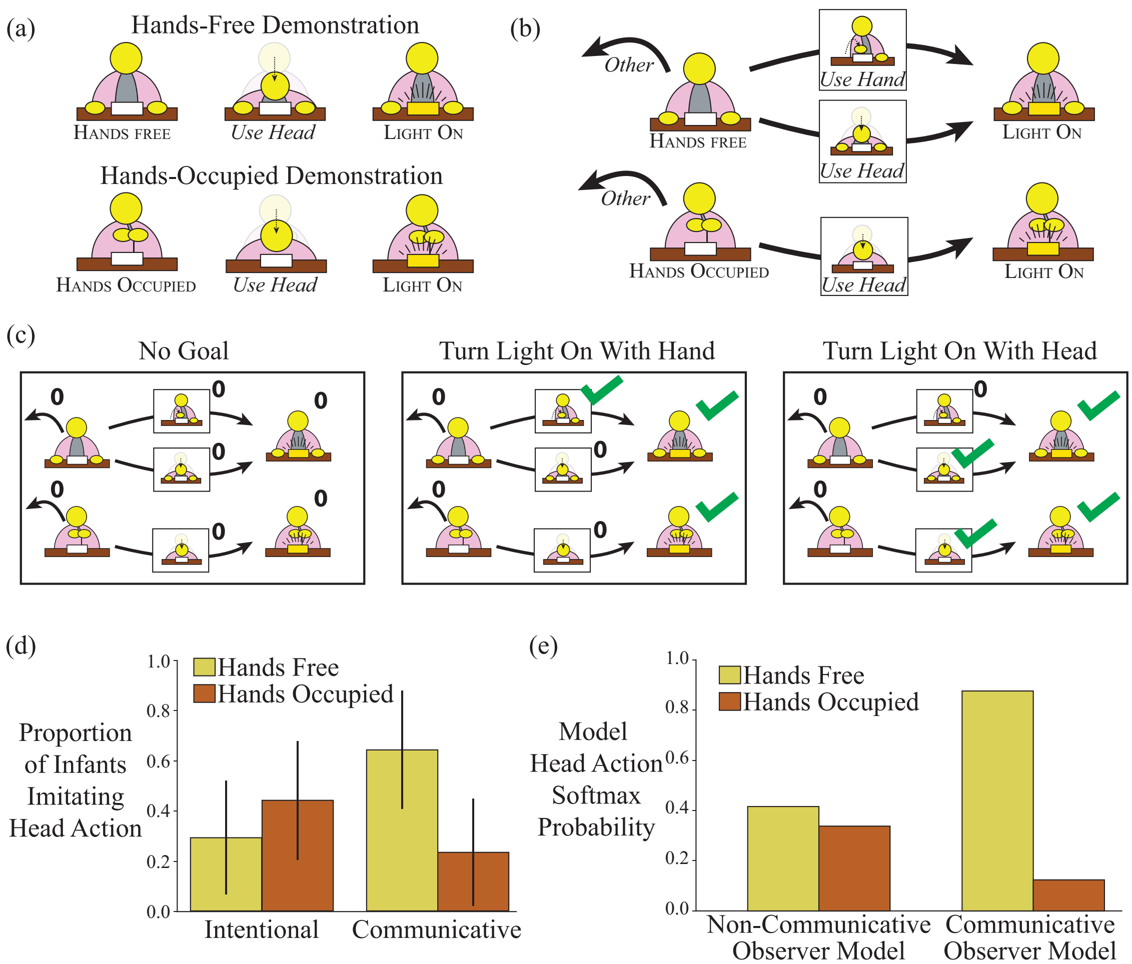

Figure 7. Summary and model of Király et al. (2013), Experiment 1 results. (a) Participants observed an experimenter demonstrate using their head to light up a novel object in one of two presentation conditions—wearing a blanket (Hands-Free) versus holding a blanket (Hands-Occupied)—and in a Communicative or Intentional condition (not shown). (b) A minimal MDP model of the environmental constraints - The demonstrator could start in the Hands Free state and deterministically transition to the Light On state by taking either *Use Head* or *Use Hand*, or take the *Other* action. Alternatively, they could start in Hands Occupied and take *Use Head* or *Other*. (c) Space of possible reward functions in model. Checks and zeros indicate +10 or 0 rewards respectively for taking an action or reaching a state. (d) Empirical results reported by Király et al. (2013). Infants differentially attempt to use their head in the Communicative conditions but not the Intentional conditions. Error bars are 95% binomial confidence intervals. (e) Model results. After observing either the hand free or hand occupied demonstration, each observer model has a belief over the three possible reward functions, which induces an expected reward function. This plots the observer's softmax probability ($\tau = 2.5$) of taking *Use Head* from Hands Free under the expected rewards. In particular, our models capture the exaggerated difference in the communicative conditions.

$\varepsilon = .05$. Additionally, to model how the infant observers *imitate* demonstrator behavior, we calculate an expected reward function (i.e. a linear combination of the three reward functions with their probabilities as weights), and report the softmax policy probabilities ($\tau = 2.5$) from the HANDS FREE state. This makes clear the relative values of actions inferred by infants that would be reflected in the head-imitation measurement.

Results. As shown in Figure 7, our model replicates the interaction between presentation condition (Hands-Free versus Hands-Occupied) and whether the demonstration is marked as communicative. In particular, compared to non-communicative observer models, the communicative observer model is more likely to imitate the head action having observed the Hands-Free demonstration and is less likely to imitate it having observed the the Hands-Occupied demonstration.

Discussion. Our model primarily explains two key findings of the results reported by Király et al. (2013): (1) Why head-use imitation increases from the Hands-Free+Intentional condition to the Hands-Free+Communicative condition, and (2) why head-use imitation decreases from the Hands-Occupied+Intentional condition to the Hands-Occupied+Communicative condition. The effects arise naturally when an observer engages in belief-directed mentalizing in the Communicative condition.

We capture the difference between the Hands-Occupied and Hands-Free conditions by formulating two parts of an MDP with analogous states but differing in whether using one's hands is available. As a result, the Hands-Free demonstrator is not merely using his head to turn on the object, he is using his head *rather than his hands*. Meanwhile, the Hands-Occupied demonstrator is not using his head *per se*, rather, he is using the *available action* to turn on the object. Our model formally captures how this difference in alternative actions influences what an observer can learn from the same sequence of actions. At the start of the experiment, the observer does not know whether the demonstrator has the goal of turning on the object, nor does she know whether the demonstrator has an unusual method for turning it on (e.g., with his head). When the demonstrator uses his head rather than his hands in the presence of an obvious alternative, this licenses the inference that using one's head is a special means. In the Hands-Occupied conditions, the observer learns about the object-directed goal to turn on the object, however, the use of one's head is “explained away” by the environmental constraint that makes the hands unavailable.

Crucially, however, the magnitude of such learning depends on whether belief-directed intentions are present: Formalizing this effect is our essential contribution. Specifically, in our model, the learner assumes that the demonstrator is engaging in belief-directed planning in the Communicative condition, but not in the Intentional condition. When reasoning about belief-directed intentions, inferences based on object-directed mentalizing become amplified. The Hands-Free demonstrator who uses her head has chosen not only to use her head, but also to select an action that is highly diagnostic of the means of using one’s head. This illustrates *inferential amplification*. Indeed, this behavior is especially diagnostic precisely because it violates a shared expectation about the efficiency of acting with one’s hands versus one’s forehead, making it also an instance of *deviation attribution*. However, when the demonstrator’s hands are occupied, this environmental constraint renders the use of his head diagnostic of an object-directed goal, but non-diagnostic of head-use as a particular means. As a result, the observer attributes the action to the belief-directed intention to convey that the novel object is a goal, rather than to convey that head-use is a means.

Learning Causal Structure from Demonstrations

Summary of Findings. Butler and Markman (2012) (Experiment 3) investigated how 3- and 4-year-olds learned about a novel causal property differentially based on the communicative context. In their paradigm, participants observe an experimenter clean up a set of objects (these had been made messy in a prior distractor tasks, ostensibly the main task). The objects included many paperclips as well as a novel object (a wooden block with tape on it) that, earlier, had been labeled a “blicket”. At the critical point of the experiment, the experimenter moves the blicket on top of the paperclips and they adhere to it by magnetic force. This “demonstration” occurred in one of three experimental conditions: Accidental, in which the blicket was apparently dropped on the paperclips while being put away and the experimenter exclaimed “Oops!”; Intentional, in which the experimenter appeared to purposefully place the blicket on the paperclips without engaging the child; and Communicative, in which he addressed the child (“Look, watch this”) before placing the blicket on the paperclips (Figure 8a). The children were then given a set of

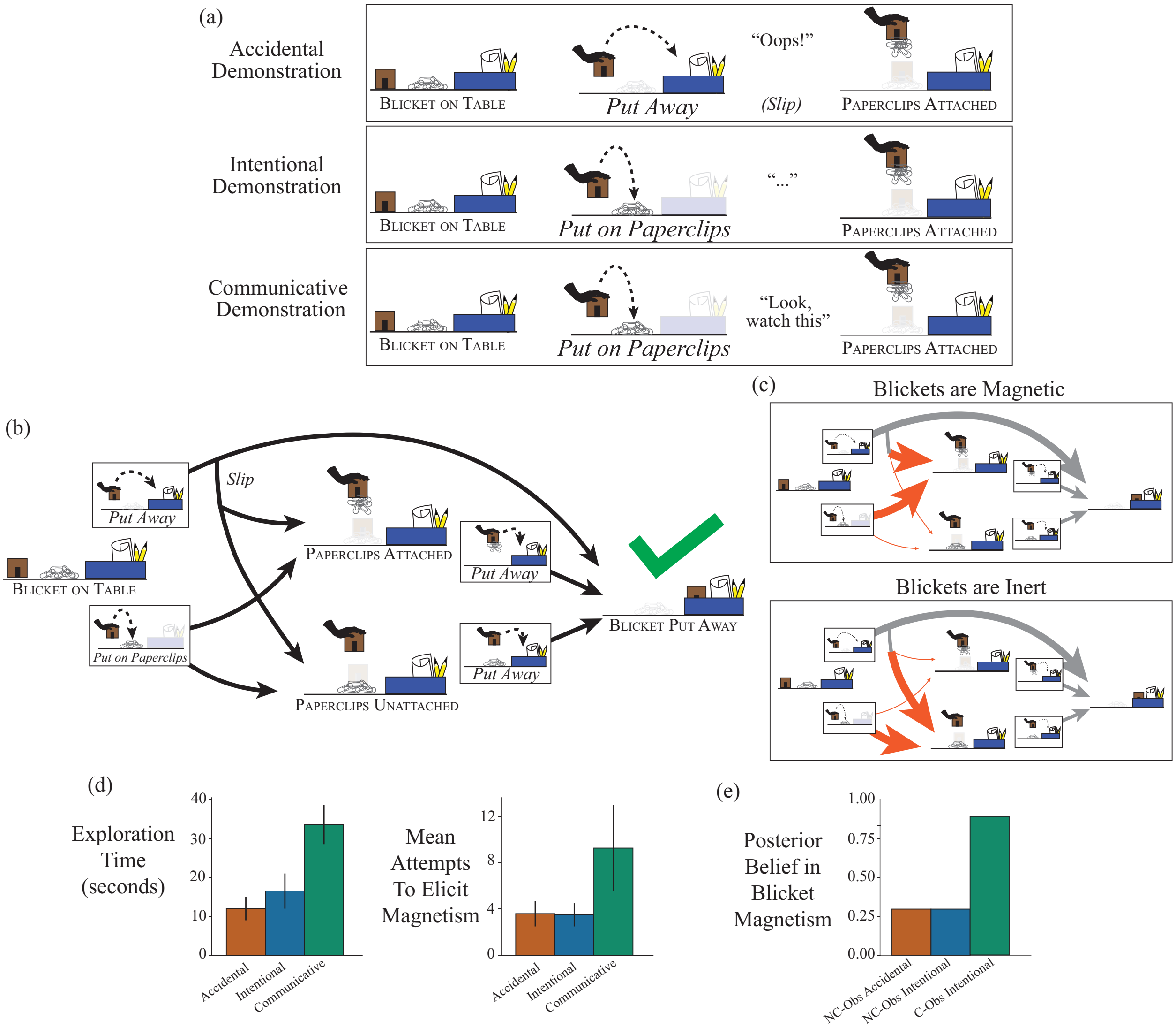

Figure 8. Summary and model of Butler and Markman (2012), Experiment 3. (a) Participants were shown a demonstration of the blicket (brown block) landing on the paperclips and sticking in one of three conditions: Accidental, Intentional, or Communicative. (b) MDP with possible transitions to and from states (Blicket on Table, Paperclips Attached, Paperclips Unattached, and Blicket Put Away) when taking actions (*Put Away* and *Put on Paperclips*). The only ground reward is putting the blicket away (green check). (c) Two possible transition functions, T_{Mag} and T_{Inert}. Arrow width indicates relative probabilities. These differ based transition probabilities to Paperclips Attached and Paperclips Unattached (highlighted in orange). (d) Empirical results. Four-year-olds explored blickets more and attempted to elicit magnetism more in the Communicative condition. (e) According to our model, a communicative context allows the observer (C-Obs) to reason explicitly about the demonstrator's belief-directed intent to convey a novel causal property, which leads to a stronger inference than when a non-communicative observer reasons only about object-directed intentions (NC-Obs).

inert (i.e., non-magnetic) blickets and paperclips to play with. Their main analyses revealed two important patterns of results. First, 4-year-olds (but not 3-year-olds) explored the blicket and attempted to use it to pick up the paperclips more in the Communicative condition compared to the other two conditions. Second, there was no detectable difference in exploration or attempts to use the inert blickets to pick up paperclips between the Accidental and Intentional conditions for 4-year-olds (Figure 8d). These results indicate an influence of communicative context on how observer inferences are drawn and shape subsequent imitation.

Intuitively, children are more likely to imitate the use of the blicket to pick up paperclips when they infer that the experimenter is communicating that this is an especially important thing to do. The principle of *inferential amplification* applies to the Communicative condition because the children interpret the intentional act of picking up paper-clips as fulfilling not only object-directed goals but also belief-directed goals. This is further accentuated by the principle of *deviation attribution* because the experimenter departs from the expected form of cleaning up in order to perform this unexpected and potentially diagnostic act. A final noteworthy feature of this experiment is that the child acquires a relevant piece of causal knowledge—the fact that the blicket can pick up paperclips, and perhaps does so reliably. The manner in which our model accommodates the transfer of causal knowledge in this case mirrors the reported experiment.

Model Formulation. To formalize the experiment described above, we first specify some assumptions about the observer's prior beliefs: (1) She knows the demonstrator has the goal of putting the blicket away; (2) she does not know whether blickets are magnetic, an unknown *causal* belief as opposed to desire; and (3) she believes that blickets are more likely to be non-magnetic than magnetic. Thus, formally, the observer starts with a distribution over two ground MDPs differing by transition function. The reward function of both places a +10 reward on having the blicket put away and a -1 step cost for each action. They differ in terms of whether blickets are inert, T_{Inert}, or magnetic, T_{Mag}. Additionally, the prior probability of them being magnetic is low (i.e., $b(T = T_{\text{Mag}}) = 0.05$).

The demonstrator starts in the state Blicket on Table and can choose from the actions *Put Away* or *Put on Paperclips*. If he chooses *Put Away*, this will most likely result in Blicket Put Away, but there is a small probability of him accidentally *slipping* and the blicket landing on the paperclips ($p_{\text{Slip}} = 0.20$). If he chooses *Put on Paperclips*, they are put on the paperclips. Depending on whether blickets are inert or magnetic, the resulting state will be either PAPERCLIPS ATTACHED or PAPERCLIPS UNATTACHED. When blickets are inert (T_{Inert}), if the blicket lands on the paperclips, the two have some small background probability of sticking to one another ($p_{\text{Background}} = 0.10$). In contrast, if the blickets are magnetic, then whether or not they stick is determined by a noisy combination of the background sticking probability and the probability that this particular blicket is magnetic ($p_{\text{Mag}} = 0.80$). Finally, whether or not the paperclips are attached to the blicket, the demonstrator can choose *Put Away* to reach the goal state, BLICKET PUT AWAY. Figure 8b shows theses states, actions, and possible transitions, while Figure 8c illustrates the relative transition probabilities associated with T_{Inert} and T_{Mag}.

This formulation of the transition dynamics allows us to distinguish between the blicket *accidentally* landing on the paperclips, which occurs in the Accidental condition, and the blicket *intentionally* landing on the paperclips, which occurs in both the Intentional and Communicative conditions (Figure 8a). The accidental demonstration can be modeled as the sequence where the demonstrator first takes the action *Put Away*, but then slips and lands on the PAPERCLIPS ATTACHED state before ending on the BLICKET PUT AWAY state. In contrast, the intentional/communicative demonstrations directly place the blicket on the paperclips by selecting *Put On Paperclips*, having them attach, and then putting it away.

All demonstrator models select actions using an ε-greedy policy ($\varepsilon = .05$). Although Butler and Markman (2012) report two measures of exploration on a different task, this is primarily in order to assess the strength of the inference about whether blickets are magnetic. Thus, we report the probabilities calculated by our model directly rather than make any assumptions about how these relate to exploratory behavior.

Results. As shown in Figure 8e, we can capture the qualitative 4-year-old results reported by Butler and Markman (2012). In particular, the final belief in blickets being magnetic is highest for a communicative observer given the intentional demonstration, which corresponds to the Communicative experimental condition. In contrast, the final belief in blickets being magnetic is lower when inference is only over object-directed intentions whether the action sequence involves the demonstrator "slipping", which corresponds to the Accidental condition, or if the demonstrator appeared to intentional place it on the paperclips, which corresponds to the Intentional condition.

Discussion. In Butler and Markman (2012), the same observation of paperclips sticking to a blicket leads to differential learning about causal structure due to communicative context. According to our account this occurs because in a communicative context the observer engages in belief-directed mentalizing.

To understand the model, first consider the Intentional condition. At the critical part of the experiment, the demonstrator has clearly indicated that his goal is to clean up the table. This means the observer is strongly disposed to interpret behavior in light of this object-directed goal. But, what inferences does this license when he intentionally places the blicket on the paperclip instead of directly putting the blicket away? In light of the demonstrator's object-directed intention to clean the table, this action appears as unexplained noise. Nonetheless, the outcome—the paperclips sticking to the blicket—suggests that blickets can pick up paperclips, albeit perhaps not reliably, since the prior belief that apparent wooden blocks attract paper-clips (or, more specifically, are sufficiently magnetic) is low.

Our model also predicts that inferences about blicket magnetism are the same in the Accidental and Intentional conditions. This is because in both conditions, the blicket landing on the paperclips is not explained by the object-directed intention to clean the table—in both cases, this act is interpreted as unexplained noise. A difference is the source of the noise: In the Intentional condition, the noise is internal to the demonstrator's decision process, while in the Accidental condition the noise is "external"; for whatever reason, the demonstrator's hand slipped. However, in both the Intentional and Accidental conditions, the paperclips stick to the blicket. Again, this outcome provides mild evidence that blickets may sometimes pick up paperclips.

In contrast, in the Communicative condition, the demonstrator indicates that he also has a belief-directed intention—he wants to show something. As a result, when the demonstrator places the blicket on the paperclips and they stick to the blicket, both the action and the outcome can be attributed to the belief-directed intention to show that blickets are magnetic. Moreover, note that deviation attribution and inferential amplification both occur here. Deviation attribution occurs because deviating from the object-directed goal of cleaning up can now be attributed to the belief-directed intention to convey that blickets are magnetic. Inferential amplification occurs because in the absence of the belief-directed intention (e.g., in the Intentional condition), there would still be a mild inference that blickets are magnetic. That this mild inference for blicket magnetism would have hypothetically occurred is strong evidence that the Communicative demonstrator actually has the belief-directed intention to convey blicket magnetism (i.e., he chose this demonstration because it was diagnostic in object-directed space), leading to amplification.

Additionally, we note that besides the behavioral results for 4-year-olds, Butler and Markman (2012) report several findings for 3-year-olds in the same experimental paradigm. First, unlike with 4-year-olds, they found exploration and attempts to use the blicket to pick up paperclips was the same between the Intentional and Communicative conditions. Second, 3-year-olds in the Accidental condition spent less time exploring and attempting to use the blicket than the other two conditions and were close to zero for both measures.

The effect of age is seemingly in conflict with other studies that found an effect of pedagogical cues in infants (e.g., Király et al. (2013) and Hernik and Csibra (2015)), but may be due to a number of factors separate from belief-directed mentalizing. For example, there were key experimental differences between experiments with children versus those with infants, such as the use of a verbal label like "blicket" or the requirement that the label be remembered throughout the experiment. Alongside developmental changes in cognitive abilities such as cognitive control or attention (Chatham, Frank, & Munakata, 2009), these factors could affect whether observers are able to track particular belief-directed intentions and their relation to other events. Although we make no specific claims about how inferential processes captured by our model will interact with the development of other cognitive abilities, our account can provide a framework for empirically investigating these interactions in more detail.

Inferring Novel Tool Functions from Demonstration

Summary of Findings. Hernik and Csibra (2015) examined how infants could learn about novel tools and their functions. In a series of studies infants observed familiarization training videos in which a demonstrator manually used a novel tool (an upside down flower pot) on an object (a banana). Their first study has two key features. First, these training

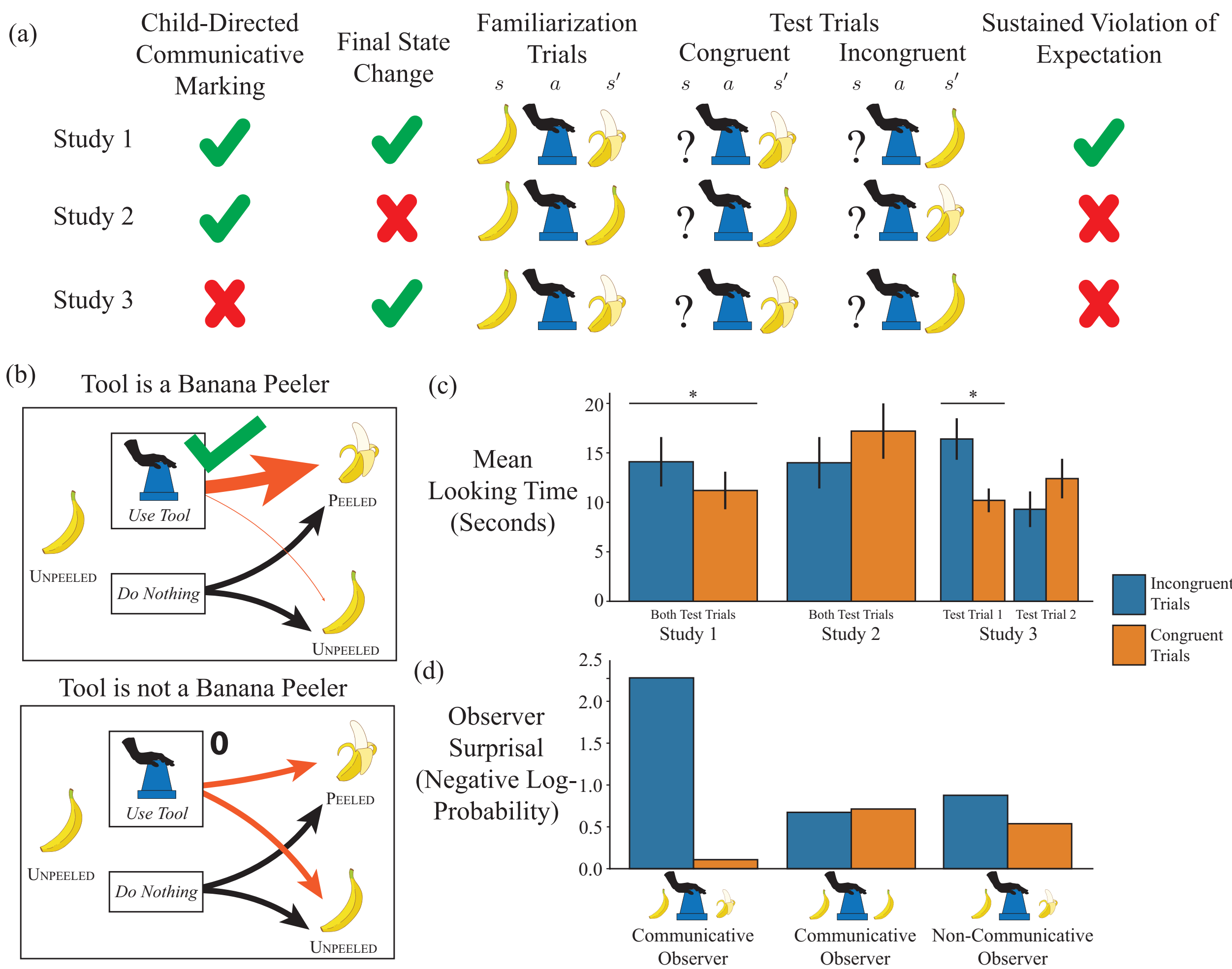

Figure 9. Summary and model of Hernik and Csibra (2015) Experiments 1, 2, and 3. (a) Infant participants viewed video trials where a novel tool was used on a banana. The banana either changed from its initial state or not when the tool was used. Violation of expectation (VOE) measures were used to assess whether a novel functional concept (e.g. the tool is a banana peeler) was learned. VOE was *sustained* across multiple test trials only when training occurred with communicative marking and target transformation. (b) Possible MDPs in our model of a single trial. The banana has a background possibility of transforming from an initial state, Unpeeled, to a new state, Peeled, regardless of what action is taken. If the tool is a banana peeler (top), then using it makes a transformation more likely and is rewarding. If it is not (bottom), then tool use has no additional effect and is not rewarding. (c) Looking-time results from studies. (d) Model posterior surprisals for congruent/incongruent test trial observations. A communicative observer trained on a tool-transformation sequence (left) has a high surprisal on an incongruent trial. The same observer model trained on a non-transformation sequence (middle) expects both sequences nearly equally. A non-communicative observer trained on a tool-transformation sequence (right) has a higher surprisal on the incongruent observation, but lower than the first communicative observer.

demonstrations were marked as communicative by the demonstrator. Second, the objects were sometimes apparently transformed by a tool (e.g. an unpeeled banana, placed briefly under a tool, became peeled). During the test trials they attempted to diagnose what the children had learned about what the tools do. In order to do this, they showed infants videos of each tool while it was in use, such that the initial condition of the object was not observed, and without any communicative marking. On *congruent* trials, the final state of an object was congruent with that of the training trials (e.g., a peeled banana). On *incongruent* trials, the resulting state was the same as the initial state typically observed in the training phase (e.g., an unpeeled banana). Critically, they found significantly greater looking times on incongruent test trials, indicating that infants' expectations were violated when the result state of the tool differed from those of the training trials (Figure 9a, first row).

The authors also reported two additional comparison studies[4]. In the first of these, demonstrations were still communicative and child-directed, however, tools *did not transform* objects in the training trials. Unlike in the original study, they found no detectable difference between congruent and incongruent test trials. In the second comparison study, demonstrations were no longer child-directed and the tools transformed the objects in the training trials (as in the first study). Here, although they found a difference in congruent and incongruent looking times for initial test trials, this difference did not persist past the first set of test trials, unlike in the original study. Collectively, these three studies (summarized in Figure 9) suggest that communicative marking *and* state changes interact when encoding novel tool functions, leading to especially robust, context-sensitive learning.

Model Formulation. Although the studies in question involve multiple counterbalanced training trials, in order to understand how the key findings relate to our account it suffices to explore the inferences our models make after observing a single training trial. Specifically, we model a trial in which the banana's initial state is UNPEELED and its final state is either PEELED or UNPEELED. Additionally, we make the following assumptions about observer prior beliefs: (1) There is a uniform background probability that the objects will change independent of tool use or

[4]Hernik and Csibra (2015) report a fourth study that conceptually replicates the results of Studies 1 and 3. The analysis of Study 4 in terms of our framework is identical to that of Studies 1 and 3.

effectiveness, and (2) arbitrary tools and arbitrary objects do not usually causally interact. Formally, this means we assume that there is a background probability of objects changing, $p_{\text{Background}} = 0.50$; that there are two relevant possible MDPs where either the banana is a peeler (M_{Peeler}) or not (M_{Inert}); and that the initial probability of the tool being a banana peeler is low (i.e. $b(M = M_{\text{Peeler}}) = 0.05$).

The demonstrator starts in the Unpeeled state and can choose either *Do Nothing* or *Use Tool*. If he chooses *Do Nothing*, then regardless of whether the tool is a banana peeler or not the state transitions to Peeled or Unpeeled according to the background probability. On the other hand, if he chooses *Use Tool*, then the probability of transitioning depends on the specific MDP. If the true MDP is M_{Peeler}, then it will transition to Unpeeled based on a combination of the background transition probability and the tool's effectiveness ($p_{\text{Effectiveness}} = .99$). Specifically, we assume that these two combine in a "noisy-or" manner where the effect occurs if either cause (or both) are activated (Pearl, 1988). Additionally, since the functional concept of a banana peeler includes a causal as well as a normative aspect that it *should* be used to peel bananas, using the tool to peel the banana gives a reward of +10 in M_{Peeler}. If the tool does not have the function of being a banana peeler and true MDP is M_{Inert}, then *Use Tool* has the same transition probabilities as *Do Nothing* and gives no reward. The different MDPs are visualized in Figure 9b.

All demonstrator models select actions with an ε-greedy policy ($\varepsilon = 0.05$). We can simulate the violation of expectation measure by calculating the *surprisal* (the negative base-2 log probability) of a congruent or incongruent trial given a model's posterior distribution. That is, each observer model is either given the transformation demonstration or non-transformation demonstration as training and produces a posterior distribution over MDPs. We can then use that distribution to calculate how surprised the model would be to see the congruent or incongruent test trials, where the actions are taken non-communicatively.

Results. Figures 9c and 9d show a qualitative correspondence between looking times for the three studies and surprisal values for the different model/training sequence combinations. First, we can compare our models of Study 1 and Study 3. In both studies, participants are trained on transformation sequences, but in Study 1 this is in a communicative context, while in Study 3 it is in a non-communicative context. In Study 1, the violation of expectation was sustained across both test trials, while in Study 3 it only occurred on the first test trial. When the observer model reasons about belief-directed intentions, it has a greater difference in surprisal between the incongruent and congruent trials, corresponding to Study 1. When it only reasons about object-directed intentions, this difference still exists but is smaller, corresponding to Study 3.

In Study 2, participants in a communicative context receive non-transformation training sequences and show no difference between the congruent and incongruent test trials. Our model of an observer reasoning about belief-directed intentions and receiving a non-transformation sequence matches this result. Specifically, the model's surprisal is almost identical for congruent and incongruent trials.

Discussion. Our model of belief-directed mentalizing explains two central features of Hernik and Csibra's results: The difference in sustained violation of expectation between Studies 1 and 3, and the absence of a violation of expectation in Study 2. According to our account, these depend on the presence or absence of inferential amplification.

First, consider the difference between Studies 1 and 3: The transformation demonstrations in Study 1 are performed in a child-directed communicative context, while those of Study 3 are not. (In other words, they correspond to the Communicative and Intentional conditions, respectively, of the papers modeled above (e.g., Király et al., 2013; Butler & Markman, 2012)). Thus, according to our model, in Study 3 participants only engage in object-directed mentalizing when interpreting the transformation sequence, which allows them to draw a weak inference that the novel object is a banana peeler, so to speak. This inference might easily be defeated given contrary evidence, however, and so it does not drive robust violation of expectation in the test phase. In contrast, in Study 1, the communicative context makes the demonstrator's belief-directed intentions apparent. As a result, the inferences based on object-directed mentalizing are amplified in light of these belief-directed intentions. In short, compared to participants in Study 3, those in Study 1 would reason that not only does the novel tool incidentally lead a change in the object, but the demonstrator wants the observer to know that this is a reliable feature of the world.

Study 2 uniquely draws out an important aspect of our model: Although a communicative context is established, the demonstrator does not perform an action with any interpretable object-directed purpose. Put simply, the tool does not do anything (alternatively, one might say, the child fully expects that a random novel tool will not peel a banana, and thus no information is conveyed). In principle, the communicative context would allow for inferential amplification, but in this case it fails because there is nothing obvious to amplify.

Notably, our model actually identifies congruent trials as slightly *more* surprising than incongruent trials because the demonstrator's continued attempts to use the tool suggests that they expect it to change, even though it does not. Indeed, although Hernik and Csibra (2015) found no significant difference between congruent/incongruent looking times, they report that more than half of the infants tended to look at the congruent events more than the incongruent events.

General Discussion

Communicative demonstrations are ubiquitous among humans, but how do they work? It is widely agreed that observers use their theory of mind to reason about the object-directed mental states of actors. In other words, they ask, "What is she trying to do, and why?" Yet, object-directed intentionality and mentalizing alone fails to account for many distinctive properties of communicative demonstrations—properties like the embellishments and exaggerations of a convincing mime act. Inspired by this observation, we show that human communicative demonstrations are best captured by a model of planning and inverse planning in an observer's belief space. In particular, our model formalizes three ideas: First, communicative demonstrators can plan over the environment as well as an observer's beliefs in order to convey information and accomplish joint object-/belief-directed goals. Second, expected transitions in an observer's belief state are grounded in object-directed theory of mind—that is, demonstrators can leverage an observer's capacity for object-directed action attribution as part of their planning model. Third, communicative observers attribute behavior to belief-directed mental states that provide information about a demonstrator's object-directed mental states. We show that a model of this form enables especially powerful observational learning in theory, and better captures data from novel and existing experimental research in practice.

Specifically, our account makes several empirical predictions that do not arise in models of object-directed intentionality and mentalizing alone. For example, compared to someone who is simply "doing" a task, someone who is "showing" how to do a task will engage in *targeted variability*, *informative inefficiencies*, and *expectation signaling* that reflect belief-directed planning. We reported several experiments in which participants could "do" or "show" tasks that differed by reward function (Experiment 1) or causal structure (Experiment 2). Compared to those who simply did tasks, participants who showed tasks engaged in precisely the kind of creative, strategic behaviors predicted by our account of planning in an observer's belief space. Model-based analyses confirmed these behavioral findings while also illustrating how the parameters of our model correspond to recoverable and interpretable measures of high-level psychological constructs.

Additionally, we examined three previously reported sets of studies of infant and child learning from communicative demonstrations. All the studies contrast learning from communicatively marked demonstrations with learning from unmarked demonstrations. These provide an ideal setting for understanding the theoretical import of our account of reasoning about belief-directed planning. At the same time, each set of studies focused on a different *type* of representation that could be learned. Király et al. (2013) examine imitation of subgoals; Butler and Markman (2012) test learning about generic causal properties; and Hernik and Csibra (2015) study inferring novel tool functions. We show how these findings can be interpreted in terms of differential inferences about object-directed and belief-directed intentions given environmental constraints, which leads to context-specific forms of *inferential amplification* and *deviation attribution* by the observer.

In short, we provide a general model for how people generate and interpret communicative demonstrations in terms of belief-directed mental states. We find extensive empirical support for this proposal in new experiments and existing findings as well as across a variety of mental representations involved in intentional action (e.g. reward functions, causal properties, subgoal structure, tool functions). The framework we developed to understand communicative demonstrations extends models of language pragmatics and pedagogy (Frank & Goodman, 2012; Shafto et al., 2014) to the setting of value-guided decision-making (Dayan & Niv, 2008; Newell &

Simon, 1972). Additionally, our formal approach provides a complementary perspective to existing accounts of the evolution and development of cognitive abilities supporting human social learning (Tomasello et al., 2005; Csibra & Gergely, 2009). In the remainder of this section, we discuss the implications of our work for formal models of social cognition as well as our understanding of the cognitive mechanisms underlying human cultural transmission, communication, coordination, and cooperation.

Relation to existing computational approaches

Our model extends existing approaches to modeling planning, social cognition, the pragmatics of language, and pedagogy. In particular, we combine aspects of each approach into a single framework that yields unique qualitative interactions that would not be predicted from any one model in isolation. Here, we discuss several ways in which our model contributes to this literature.

Intentional Action as a Message-to-Signal Mapping. Our model reveals deep connections between communicative demonstrations and other forms of communication, such as language. In Rational Speech Act (Frank & Goodman, 2012; Goodman & Frank, 2016; Yoon, Tessler, Goodman, & Frank, 2017) and Bayesian Pedagogy (Shafto et al., 2014) models, a transmitter (e.g. a speaker or teacher) provides a signal (e.g. an utterance or an example) to a receiver (e.g. a listener or learner) who must infer an underlying message (e.g. a linguistic meaning or novel concept)[5]. In these settings, given a base *message-to-signal mapping*, recursive social reasoning amounts to the transmitter and receiver recursively anticipating each others' selection of a message and interpretation of that message (up to a finite or infinite depth). In Rational Speech Act models, the base mapping is the semantics of words, while in Bayesian Pedagogy models, the base mapping is often a probabilistic concept class. One way to identify a base mapping is to estimate it empirically, for instance, by asking people what the expected semantics are in non-pragmatic contexts (Frank & Goodman, 2012; Kao et al., 2014). Alternatively, one can derive constraints on a base mapping given that optimal cooperative inference is possible (Yang et al., 2017). Regardless, from a theoretical perspective, the selection of a base mapping is critical since it directly determines the refined mapping that results from a recursive reasoning process.

Here, we describe an additional form of base-mapping: an agent's possible preferences and representations of the environment (i.e., desires and beliefs). In our model, the base "message-to-signal" mapping is from possible worlds that a demonstrator could be acting within to likely intentional actions in that world. That is, the base mapping is derived from value-guided decision-making, a general framework for describing the behavior of any adaptive system or organism (Sutton & Barto, 1998; Friston, 2010; Newell, 1982; Anderson, 1990). Prior work has established that the capacity to *recognize* intentional behavior is present in humans from a young age (Gergely & Csibra, 2003; Malle, 2008). In recent years, various aspects of intention-reading, including reasoning about beliefs, desires, intentions, uncertainty, and emotions, have been cast as inverse planning (Baker et al., 2017; Kiley Hamlin, Ullman, Tenenbaum, Goodman, & Baker, 2013; Ong, Zaki, & Goodman, 2015), which we utilize in this work. Put simply, we propose that people can ground an inference about what a person is trying to communicate in an inference about what she is trying to do. This use of inverse planning as a base message-to-signal mapping has a well-established theoretical and empirical basis in the existing literature.

Planning and Inverse Planning over a Joint Model. Our model treats generating communicative demonstrations as choosing a sequence of actions that solve a particular *planning* problem (Newell & Simon, 1972; Puterman, 1994; LaValle, 2006). In particular, we assume that planning not only occurs over the ground state space but also in an *observer's belief space*. And, we assume that observers anticipate and reason about these joint goals. In other words, planning occurs over a composition of a world and observer belief model, which enables reasoning about simultaneous ground and belief transitions as well as their payoffs. Interpreting communicative demonstrations, in turn, relies on inverse planning over this joint model.

Our formulation of communicative demonstrations as planning and inverse planning over a joint world-belief model complements previous computational approaches in several ways. For example, Buchsbaum et al. (2011) and Shafto et al. (2014) (Experiment 3) investigate teaching with and learning from multiple actions or interventions on a causal system. Our account further

[5]We borrow generic terms such as "transmitter", "signal", "receiver", and "message" from information theory (Shannon & Weaver, 1949)

develops this idea by modeling actions as being part of a unified plan or intention, which closely relates to our use of object-directed intentionality as a base signal-to-message mapping.

Another related approach to our own is reflected in work by Rafferty et al. (2016), who formulate selecting sequences of teaching actions during concept learning as a partially observable MDP-planning problem. The domains they examine involve “pure” belief planning, whereas our setting requires extending this to consider how a belief transition model derives from and composes with a ground world model.

Finally, models of language pragmatics show how speakers and listeners can coordinate on a single communicative goal for a single utterance (Frank & Goodman, 2012; Kao et al., 2014). An important extension of this considers how tradeoffs between communicative goals and social goals explain polite speech (Yoon et al., 2017). This resembles our approach of modeling object-level and belief-level trade-offs, although it differs in that we also treat object-directed intentionality as a base mapping for belief-directed intentionality attribution.

The differences we have noted between our model and previous work reflects the distinctive challenges of synthesizing communication and value-guided decision-making in a unified framework. Many past models consider the problem of communication in more pure settings, where object-directed planning is less relevant (Frank & Goodman, 2012; Shafto et al., 2014). But a successful theory of communicative demonstration must explain how people both communicate goals, beliefs and obstacles while also planning to accomplish goals given beliefs and obstacles. On the one hand, integrating these two different objectives makes planning and inverse planning complicated; on the other hand, grounding communication in object-directed intentionality makes it quite efficient and flexible. By treating communicative demonstrations as a planning and inverse planning problem over a joint world-belief model, we can capture these interactions in a unified manner.

Communication as Sequential Planning. By describing communicative demonstrations as planning and inverse planning over object-belief states, insights from general formulations of planning can be applied. Conveying complex, multi-faceted mental structures like skills, domain knowledge, or systems of norms relies on systematically communicating simpler components. Mechanisms originally posited to explain how people successfully engage in large-scale problem solving could also play a key role in how people solve the "problem" of communicating. These include hierarchical action representations (Botvinick, Niv, & Barto, 2009; Barto & Mahadevan, 2003), state abstractions (Li, Walsh, & Littman, 2006), and function approximation (Sutton, McAllester, Singh, & Mansour, 2000). At the same time, this account accommodates how belief-directed subgoals could play an important part in hierarchical planning and ongoing interaction. For instance, suppose a parent has the goal of their son regularly doing chores around the house and formulates a plan to accomplish this. A belief-directed subgoal in this plan would include showing him how to vacuum, which itself could be broken down into further belief-directed subtasks like showing him where to vacuum or how to replace the filter.

Our account allows communicative demonstrations to inherit many of the theoretical and methodological tools used in planning and problem-solving. This also allows us to consider how belief-directed planning interacts with standard planning mechanisms. For instance, do the two forms of planning rely on the same underlying cognitive machinery? Or are there specialized processes for belief-directed planning? Are there specific representations or abstractions that facilitate planning and inverse planning in communicative settings? Future work will need to investigate such questions.

Proximal Mechanisms of Human Sociality

Human sociality is distinctive in both degree and kind. We teach, learn, communicate, cooperate, compete, and coordinate in amounts and ways rarely observed in other species. Part of the explanation for this is simple: It is evolutionarily adaptive. For example, computational models of cultural evolution have demonstrated how social learning strategies such as imitation give rise to cumulative culture and can lead to higher overall fitness in a species (Boyd & Richerson, 1995; Thompson, Kirby, & Smith, 2016). Similarly, evolutionary models of cooperation and coordination establish how costly signaling behaviors can emerge as equilibrium strategies in games with replicator-dynamics (Gintis, Smith, & Bowles, 2001; Jordan, Hoffman, Bloom, & Rand, 2016; Jordan, Hoffman, Nowak, & Rand, 2016; Hoffman, Yoeli, & Nowak, 2015). However, a full account of human sociality must also explain why humans exhibit these cultural learning and signaling behaviors in ways that other animals do not. A number of proposals address the question of what proximal mechanisms support human sociality by emphasizing the role of particular forms of mentalizing and theory of mind (Tomasello et al., 2005; Csibra & Gergely, 2009; Tennie, Call, & Tomasello, 2009; Lewis & Laland, 2012; Shneidman & Woodward, 2016; Ho, MacGlashan, Littman, & Cushman, 2017). Here, we discuss how our formal account relates to questions raised and phenomena identified in these debates as well as how it provides insight into future research directions.

Social learning. Many simple models of social learning propose that people "copy others' behaviors". But what, exactly, does this mean? Presumably the behavior is not represented at the level of the most fine-grained motor commands ("First, move your hand four inches at an angle of 15° and then fully flex your fingers..."), but rather in terms of abstract goals and subgoals ("First, grasp the cup..."). Thus, in order to copy other's behaviors, it is necessary to infer unobservable mental state variables, such as goals and beliefs, from observable actions.

A major challenge immediately arises, however: In most real-world settings, actions are indeterminate. If you observe someone kick a soccer ball in a soccer game, what are their goals and beliefs? Were they planning to score a goal? Make a pass? Show off how well they kick? Give the ball to the other team in order to fix the game? Identifying mental states from action is an ill-posed problem because the same behavior can fall under different descriptions (Anscombe, 1963), their relationship to mental states is opaque (Csibra & Gergely, 2009), and we are often uncertain about the complex environment in which the action occurred.

The kind of communicative demonstrations that we model help to reduce this indeterminacy, and thus to make social learning a more tractable problem. When a demonstrator and observer are coordinated in their construal of an demonstration as both a object-directed and belief-directed episode, this can greatly sharpen the precision with which the observer infers the beliefs and goal of the demonstrator. For instance, if I want you to learn how to pass the ball to a teammate (as opposed to learn something else), I can modify my behavior to optimize your learning of that idea specifically. Our account develops this idea by formulating how people can leverage others' capacity for theory of mind to convey mental structure through belief-directed planning. The demonstrator's attitude towards the observer then sets in motion a virtuous cycle whereby observers can then interpret behavior in terms of object-directed and belief-directed intentions, which further enhances the fidelity of intentional signaling behaviors.

Signaling. Although we have described many examples of communicative demonstration aimed at teaching skills, the model itself applies to a much larger array of human behaviors. Often, for instance, we use communicative demonstrations to convey our feelings (e.g., giving roses on Valentine's Day), intellect (e.g., asking a very technical question during a department colloquium) or income (e.g., driving a Maserati). Such demonstrations often rely on costly signaling (Gintis et al., 2001), which arises in social dilemmas involving information search (Hoffman et al., 2015), time-consuming deliberation (Jordan, Hoffman, Nowak, & Rand, 2016; Levine, Barasch, Rand, Berman, & Small, 2018), and third-party punishment (Millet & Dewitte, 2007; Fehrler & Przepiorka, 2013; Jordan, Hoffman, Bloom, & Rand, 2016). Such behaviors are adaptive because they signal important information to social partners; for instance, third-party punishment signals trustworthiness (Jordan, Hoffman, Bloom, & Rand, 2016).

Although it has sometimes been overlooked, the action indeterminancy problem described above is also a key constraint on signaling behaviors. That is, the literal interpretation of many actions may be too ambiguous to serve as effective signals. This means that the range of signaling-based strategies that could sustain cooperation will be inherently limited by the fidelity of the action generation/interpretation channel. The "base" mapping from actions to general characteristics may have enough fidelity for completely unambiguous, pre-programmed signaling strategies to work. But, as in the case of cumulative cultural evolution, more sophisticated signaling-based strategies may only be accessible given the cognitive machinery for context-sensitive communicative planning and interpretation. The capacity for adaptive planning and inverse planning over beliefs can thus serve as a high-fidelity communication mechanism that overcomes this constraint.

Explaining the uniqueness of human sociality. Non-human animals are capable of both social learning (Whiten et al., 1999) and costly signaling (Gintis et al., 2001), but on a vastly diminished scale and with far less flexibility than humans (Boyd, Richerson, & Henrich, 2011). We suggest that this may owe to a human-unique capacity for planning and inverse planning in belief space.

While our closest animal relatives are highly social and have basic theory of mind abilities, current evidence suggests that they do not engage in the kind of explicit, active, belief-directed planning we model here. That is, while they track and respond to others' mental states, they do not attempt to directly manipulate them. For example, younger chimpanzees can learn how to use objects as tools from adults via imitation, which allows chimpanzee societies to sustain local tool-based traditions and cultures (Whiten et al., 1999). However, there is little evidence that adults actively teach their young how to use tools, a clearly adaptive opportunity to apply belief-directed planning (Thornton & Raihani, 2008; Lewis & Laland, 2012). Similarly, although non-human primates understand what others can see, have seen, and how perception should influence behavior (Hare et al., 2001; Krupenye et al., 2016), there is little evidence that they ever actively attempt to influence others' perception to transmit information (Tomasello et al., 2005). Finally, although there is evidence that chimpanzees can acquire "intention-movement" signaling gestures that are functionally similar to communicative demonstrations (Tomasello, 2010; Call & Tomasello, 2007), these seem to arise through associative or model-free learning mechanisms (Dayan & Niv, 2008) rather than the planning processes we model.

The proposal that non-human primate demonstrators do not engage in belief-directed planning can explain one of the most noteworthy empirical findings in the comparative literature on social learning. While there is now an emerging consensus that both children and chimpanzees can imitate in the strong sense of copying intentions (Whiten et al., 2009), only children engage in "over-imitation." A typical over-imitation experiment first involves a demonstrator achieving a goal (e.g. retrieving candy from a closed transparent box) by taking a sequence of apparently causally irrelevant (e.g. tapping on the box with a stick) and relevant actions (e.g. opening the box). After seeing this sequence, human children will imitate the entire sequence, including the irrelevant actions, whereas non-human primates only imitate the relevant actions (Lyons, Damrosch, Lin, Macris, & Keil, 2011; Keupp, Behne, & Rakoczy, 2018; Clay & Tennie, 2018). Although this "blind copying" by human children makes adaptive sense in the context of high-fidelity transmission, at the proximal level of individual cognition, over-imitation can appear to be a kind of cognitive error.

Our model provides a coherent account of over-imitation in rational terms that elaborates on previous proposals (e.g., Buchsbaum et al., 2011). A key phenomenon reflected in our models is that an observer who only reasons about object-directed intentions will treat strange actions as noise, whereas one that even minimally considers the possibility of belief-directed intentions will attribute strange actions to them and by extension some underlying mental structure. This is *deviation attribution*. Moreover, our account suggests that, to a sophisticated observer, it is precisely because some actions *appear unnecessary* relative to background knowledge about how the world works that, they are *actually necessary* for some non-obvious, action-relevant reason (e.g. hidden causal machinations, social/moral norms, in-group practices, etc.). Meanwhile, a naïve observer who only reasons about object-directed intentions can only attribute the strange actions they see to inexplicable noise. In light of this result, the specifically human tendency for over-imitation falls naturally out of the suggestion that humans may have a unique capacity for planning and inverse planning in observer belief space (even while the capacity for planning and inverse planning in object space is more taxonomically extensive).

Several elements of this proposal, however, require further investigation. For example, we need stronger tests of the absence of belief-directed planning in non-human primates. Additional over-imitation experiments with children that manipulate the presence of communicative intent given different contexts would also be needed. But, if belief-directed planning and inverse planning is a key explanatory factor, then why? One possibility is that non-humans lack a motivation to share mental states (Tomasello et al., 2005). Another possibility is that belief-directed planning and inverse planning entail computational demands beyond the capacity of non-humans.[6]Future work that combines computational tools with human and ape studies can help provide answers to such questions.

Prior Expectations of Communicative Actions and Intentions. Like previous computational accounts of pragmatics and pedagogy (Frank & Goodman, 2012; Shafto et al., 2014), we model communicative observers as Bayesian reasoners who perform inference given a demonstration. Put another way, our model conceptually separates an observer's *prior beliefs* about object- and belief-directed mental states and the *generative process* of a demonstrator's planning. Although our discussion has largely emphasized the planning process, the prior beliefs

[6]For instance, see Van Rooij et al. (2011) for an analysis of the computational difficulty of intentional communication of the form we model here.

clearly plays a critical role. For instance, ostensive cues (such as saying a child's name, or making eye contact) can be understood as modifying prior expectations by signaling the presence of belief-directed intentions. This is precisely how we model the findings of Király et al. (2013); Butler and Markman (2012); Hernik and Csibra (2015), where a key experimental manipulation is whether there is an ostensive cue directed towards the participant. Might efficient coordination on ostensive cues between demonstrator and observer require prior knowledge—an expectation that saying a name, or making eye contact, is a communicatively-relevant act?

As this example illustrates, several important questions in human social learning can be cast as questions about the nature of priors over communicative goals. For example, the theory of natural pedagogy (Csibra & Gergely, 2009) emphasizes that the recognition of certain stimuli as ostensive cues is innate, whereas others have argued for that their meaning is acquired through immersion in a particular cultural and social environment (Shneidman & Woodward, 2016). Our account cannot settle this debate but, because it parallels debates in other cognitive domains, we can draw insights from formal models of those domains. For example, the abstract notion of causality has been proposed to be innate (Hume, 1748), but computational work by Goodman, Ullman, and Tenenbaum (2011) demonstrates how it can be just as efficient to learn an abstract theory of causality in conjunction with specific causal models. An analogue of this kind of efficient hierarchical learning may occur in the context of learning ostensive cues based on general theory of mind representations.

Natural pedagogy also proposes that infant and child observers automatically treat communicative demonstrations as conveying *relevant, generalizable information* (e.g. "blickets are magnetic") (Csibra & Gergely, 2009), and a number of findings support this view (Butler & Tomasello, 2016; Butler & Markman, 2012). This claim can also be understood as involving priors over others' communicative goals, which allows us to ask why infants and children would have certain priors. For example, in the context of cultural transmission, social learning, and larger decision-making processes, it makes sense for communicative intentions to be about generic information: Adults already have mental structures, including generic knowledge of artifacts and natural kinds, that are adapted to the world; infants and children generally lack this information if only because they have less experience (Ho et al., 2017). Put another way, given the experiential asymmetry between adults and children, generic information is relevant.

Additionally, our formalization naturally explains how an observer's initial beliefs about the world themselves could influence expectations about communicative intentions. For example, in an experiment by Southgate et al. (2009), children imitated actions instead of goals depending on whether they already had received information about goals. From an observer's perspective, this could be explained in terms of expected communicative intentions being coupled to one's own beliefs—e.g. an observer expects communicative intentions to be about new information. Importantly, our formulation allows us to precisely distinguish between how an observer's belief state affects what communicative intentions are going to be about (i.e. the prior) from how belief states are intentionally modified given a communicative goal (i.e. the likelihood). Future computational and developmental work will be needed to elucidate the interaction of these two features of communicative demonstrations as well as other contextual effects on priors.

Demonstration as a bridge to language. Our account takes inspiration from prior treatments of linguistic communication such as language pragmatics (Goodman & Lassiter, 2015). But our treatment of communicative demonstrations can also be used to formalize the relationship between pre-linguistic and linguistic communication explored by previous researchers (e.g., Tomasello et al., 2005; Sperber & Wilson, 1986). Specifically, we model communicative demonstrations as deriving from recursive social reasoning grounded in an individual agent's intentional action, which helps explain how they work as a form of non-symbolic communication. In contrast, language is quintessentially symbolic; linguistic symbols can be arbitrary, stand in for abstract concepts, and combine combinatorially (De Saussure, 1916; Chomsky, 1957; Montague, 1970; Jackendoff, 2003). However, like communicative demonstrations, language is used to accomplish belief-directed goals since they communicate ideas and seemingly "program" other minds (Lupyan & Bergen, 2016). This suggests a continuity from non-symbolic processes to language. Possibly, then, communicative demonstrations provide a developmental or phylogenetic bridge from the individual, cognitive processes involved in value-guided decision-making to the interpersonal, symbolic processes exemplified by linguistic communication.

The connection from decision-making to symbols has several implications. First, the evolution of language may have been reliant on the capacity to engage in higher-order theory of mind reasoning initially present in communicative demonstrations. This would explain the co-occurrence of language and complex forms of communicative sociality in humans and their relative absence in non-human primates (Whiten et al., 2009; Tomasello et al., 2005). Second, the acquisition of language and symbol use can piggyback on non-symbolic communicative behaviors we analyze here. Accounts that link language learning with intentional and pragmatic cues as well as play could be better understood in light of our framework (Tomasello, 2000; Weisberg, Zosh, Hirsh-Pasek, & Golinkoff, 2013). It is also consistent with accounts of communicative demonstrations of object properties serving as non-verbal generic symbols in pre-verbal infants (Csibra & Shamsudheen, 2015). Finally, it suggests how new symbols can be created and become part of a linguistic system: Novel shared problem-solving settings create the need for individuals to convey mental structure to one another, which can be initially accomplished through non-symbolic mechanisms like communicative demonstrations. This can then scaffold the assignment of meaning to new symbols by making the need for a novel symbol apparent or via processes of conventionalization (Wittgenstein, 1953; Scott-Phillips & Kirby, 2010; Theisen, Oberlander, & Kirby, 2010). Continued computational and empirical research into the cognitive mechanisms underlying communicative demonstrations can help elucidate the relationships between value-guided decision-making and language.

Conclusion

We have formulated and tested a computational account of communicative demonstrations based on rational planning and inverse planning over belief states. The model we develop builds on existing theoretical work and is supported by the results of novel experiments and previously reported findings. This account provides insight into the mechanism of human communication, imitation, and interaction while also suggesting future directions for better understanding how communicative demonstrations relate to other dimensions of human social cognition.

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Appendix A

Softmax-ε optimal policies

Following Luce (1959); Nassar and Frank (2016); Collins and Frank (2018), we model deviations from perfect optimality as:

$$\pi_i^{\mathrm{Do}}(a_t \mid w_t) = (1 - \varepsilon^{\mathrm{Do}}) \frac{\exp\{Q_i(w_t, a_t)/\tau^{\mathrm{Do}}\}}{\sum_{a' \in \mathcal{A}(w_t)} \exp\{Q_i(w_t, a')/\tau^{\mathrm{Do}}\}} + \varepsilon^{\mathrm{Do}} |\mathcal{A}(w_t)|^{-1}, \tag{5}$$

where $\tau^{\mathrm{Do}} > 0$ is a softmax temperature parameter and $\varepsilon^{\mathrm{Do}} \in [0, 1]$ is a small probability of choosing an action uniformly at random.

Appendix B

Implementation Details

Approximating belief-space policies

Fitting parameters from an observer belief MDP, M_{Obs}, to participant behavior requires calculating a softmax probability based on its value function. Since the belief space is continuous and exact methods such as those for POMDPs do not apply, it is necessary to approximate the value function. We did this by constructing a discretized MDP, $\tilde{M}_{\text{Obs}}$ (Munos & Moore, 2002). We discretized the original belief-state space to a set S_D and constructed a transition function where, for each $a \in A$ and $s_D \in S_D$, $\tilde{T}(s'_D \mid a, s_D) = \sum_{s'} T_{\text{Obs}}(s' \mid a, s_D) NN(s', s'_D)$, where $NN(s', s'_D)$ is an indicator function for whether s'_D is the nearest neighbor of s' within S_D. The set S_D itself was constructed by selecting evenly spaced points over the n-dimensional space. Each dimension was split into b bins. Additionally, to increase the resolution of the value function for belief states actually visited by participants, we used the data from all trials and both conditions to calculate the belief states that would have been visited given they were taking actions in M_{Obs}. We then use value iteration to calculate the exact value function in this discretized MDP and obtain softmax probabilities.

Appendix C

Modeling Details

Parameter	Values
γ^{Do}	.8, .85, .9, .95, .99, .9999
ε^{Do}	0.0, .025, .05, .075, .1, .125, .15, .175, .2
τ^{Do}	0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0
γ^{Show}	.8, .85, .9, .95, .99
$\varepsilon^{\text{Show}}$	.01, .02, .03, .04, .05, .06, .07, .08, .09, .1, .2, .3
τ^{Show}	0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.75, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0
κ	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25

Table C1

Experiment 1: Model-parameters searched in gridsearch.

Parameter	Values
γ^{Do}	.1, .2, .3, .4, .5, .6, .7, .75, .8, .85, .9, .95, .99
ε^{Do}	0.0, .02, .06, .08, .12, .16, .18, .22, .26, .28, .32, .36, .38, .42, .46, .48
τ^{Do}	0.00, 0.05, .1, 0.15, .2, .25, .3, .35, .4, .45, .5, .55, .6, .65, .7, .75, .8, .85, .9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0
γ^{Show}	.1, .2, .3, .4, .5, .6, .7, .75, .8, .85, .9, .95, .99
$\varepsilon^{\text{Show}}$	0.0, .02, .06, .08, .12, .16, .18, .22, .26, .28, .32, .36, .38, .42, .46, .48
τ^{Show}	0.00, 0.05, .1, 0.15, .2, .25, .3, .35, .4, .45, .5, .55, .6, .65, .7, .75, .8, .85, .9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0
κ	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25

Table C2

Experiment 3: Model-parameters searched in gridsearch.